Finite Element Methods

In general, finite element methods are used to solve partial differential equations in two or three space variables and are widely used to solve fluid flow problems. The method involves subdividing a large fluid flow domain into smaller, simpler parts that are called finite elements. This usually involves space discretization that is implemented by the construction a mesh or grid extending over most if not all of the flow field and involves a finite number of grid points. A set of simple equations are proposed to model the variations in the flow properties over each of the elements and these are substituted into the partial differential equations for the flow to obtain a set of algebraic equations for each of the elements that include all the individual, as-yet-undetermined parameters used in constructing the simple equations. Those equations are then assembled into a larger system of equations that model the entire problem. The finite element method then uses variational methods to approximate a solution by minimizing an error function associated with the system of algebraic equations and thus determining the parameters.

A discretization strategy is understood to mean a clearly defined set of procedures that cover (a) the creation of finite element meshes, (b) the definition of basis function on reference elements (also called shape functions) and (c) the mapping of reference elements onto the elements of the mesh. Examples of discretization strategies are the h-version, p-version, hp-version, x-FEM, isogeometric analysis, etc. Each discretization strategy has certain advantages and disadvantages. A reasonable criterion in selecting a discretization strategy is to realize nearly optimal performance for the broadest set of mathematical models in a particular model class.

The various numerical solution algorithms can be classified into two broad categories; direct and iterative solvers. These algorithms are designed to exploit the sparsity of matrices that depend on the choices of variational formulation and discretization strategy. A common form of the variational formulation is the Galerkin method in its various forms that include earlier methodologies such as the Rayleigh-Ritz method.

Postprocessing procedures are designed for the extraction of the data of interest from a finite element solution. In order to meet the requirements of solution verification, postprocessors need to provide for a posteriori error estimation in terms of the quantities of interest. When the errors of approximation are larger than what is considered acceptable then the discretization has to be changed either by an automated adaptive process or by the action of the analyst. There are some very efficient postprocessors that provide for the realization of superconvergence.

For further information on finite element methods for the solution of fluid flows, the reader is referred to the extensive finite element literature and internet resources.