Averaging

As discussed in section (Nba), when the ratio of the particle size, D, to the typical dimension of the averaging volume (estimated as the typical length, ϵ, over which there is significant change in the averaged flow properties) becomes significant, several issues arise (see Hinze 1959, Vernier and Delhaye 1968, Nigmatulin 1979, Reeks 1992). The reader is referred to Slattery (1972) or Crowe *et al.* (1997) for a systematic treatment of these issues; only a summary is presented here. Clearly an appropriate volume average of a property, Q_C, of the continuous phase is given by $< Q_C >$ where

$$< Q_C >= \frac{1}{V_C} \int_{V_C} Q_C dV \tag{Ndb1}$$

where V_C denotes the volume of the continuous phase within the control volume, V. For present purposes, it is also convenient to define an average

$$\overline{Q_C} = \frac{1}{V} \int_{V_C} Q_C dV = \alpha_C < Q_C > \tag{Ndb2}$$

over the whole of the control volume.

Since the conservation equations discussed in the preceding sections contain derivatives in space and time and since the leading order set of equations we seek are versions in which all the terms are averaged over some local volume, the equations contain averages of spatial gradients and time derivatives. For these terms to be evaluated they must be converted to derivatives of the volume averaged properties. Those relations take the form (Crowe *et al.* 1997):

$$\overline{\frac{\partial Q_C}{\partial x_i}} = \frac{\partial \overline{Q_C}}{\partial x_i} - \frac{1}{V} \int_{S_D} Q_C n_i dS \tag{Ndb3}$$

where S_D is the total surface area of the particles within the averaging volume. With regard to the time derivatives, if the volume of the particles is not changing with time then

$$\overline{\frac{\partial Q_C}{\partial t}} = \frac{\partial \overline{Q_C}}{\partial t} \tag{Ndb4}$$

but if the location of a point on the surface of a particle relative to its center is given by r_i and if r_i is changing with time (for example, growing bubbles) then

$$\overline{\frac{\partial Q_C}{\partial t}} = \frac{\partial \overline{Q_C}}{\partial t} + \frac{1}{V} \int_{S_D} Q_C \frac{Dr_i}{Dt} dS \tag{Ndb5}$$

When the definitions (Ndb3) and (Ndb5) are employed in the development of appropriate averaged conservation equations, the integrals over the surface of the disperse phase introduce additional terms that might not have been anticipated (see Crowe *et al.* 1997 for specific forms of those equations). Here it is of value to observe that the magnitude of the additional surface integral term in equation (Ndb3) is of order $(D/\epsilon)^2$. Consequently these additional terms are small as long as D/ϵ is sufficiently small.