Kinematic Shock Stability

The stability of the kinematic shock waves analyzed in the last section is most simply determined by considering the consequences of the shock splitting into several fragments. Without any loss of generality we will assume that component A is less dense than component B so that the drift flux, j_{AB}, is positive when the upward direction is defined as positive (as in figure 1, section (Nse), and figure 1, section (Nsb)).

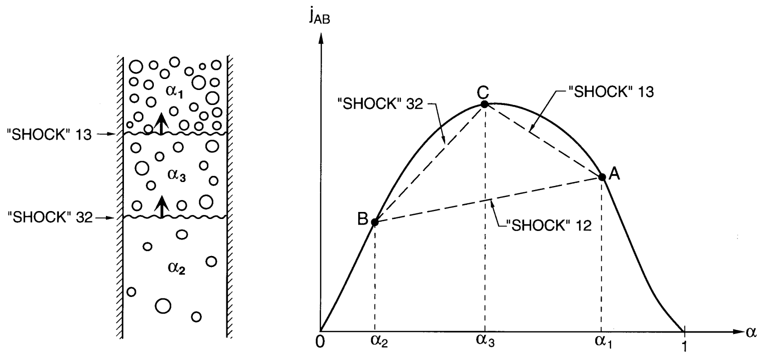

Figure 1: Shock stability for $\alpha_1 > \alpha_2$.

Consider first the case in which $\alpha_1 > \alpha_2$ as shown in figure 1 and suppose that the shock begins to split such that a region of intermediate volume fraction, α_3, develops. Then the velocity of the shock fragment labeled *Shock* 13 will be given by the slope of the line CA in the drift flux chart, while the velocity of the shock fragment labeled *Shock* 32 will be given by the slope of the line BC. The former is smaller than the speed of the original *Shock* 12 while the latter fragment has a higher velocity. Consequently, even if such fragmentation were to occur, the shock fragments would converge and rejoin. Another version of the same argument is to examine the velocity of small perturbations that might move ahead of or be left behind the main *Shock* 12. A small perturbation that might move ahead would travel at a velocity given by the slope of the tangent to the drift flux curve at the point A. Since this velocity is much smaller than the velocity of the main shock such dispersion of the shock is not possible. Similarly, a perturbation that might be left behind would travel with a velocity given by the slope of the tangent at the point B and since this is larger than the shock speed the perturbation would catch up with the shock and be reabsorbed. Therefore, the shock configuration depicted in figure 1 is stable and the shock will develop a permanent form.

On the other hand, a parallel analysis of the case in which $\alpha_1 < \alpha_2$ (figure 2), clearly leads to the conclusion that, once initiated, fragmentation will continue since the velocity of the shock fragment *Shock* 13 will be greater than the velocity of the shock fragment *Shock* 32. Also the kinematic wave speed of small perturbations in α_1 will be greater than the velocity of the main shock and the kinematic wave speed of small perturbations in α_2 will be smaller than the velocity of the main shock. Therefore, the *shock* configuration depicted in figure 2 is unstable. No such shock will develop and any imposed transient of this kind will disperse if $\alpha_1 < \alpha_2$.

Using the analogy with gas dynamic shocks, the case of $\alpha_1 > \alpha_2$ is a compression wave and develops into a shock while the case of $\alpha_1 < \alpha_2$ is an expansion wave that becomes increasingly dispersed. All of this is not surprising since we defined A to be the less dense component and therefore the mixture density decreases with increasing α. Therefore, in the case of $\alpha_1 > \alpha_2$, the lighter fluid is on top of the heavier fluid and this configuration is stable whereas, in the case of $\alpha_1 < \alpha_2$, the heavier fluid is on top and this configuration is unstable according to the Kelvin-Helmholtz analysis (see section (Njo)).

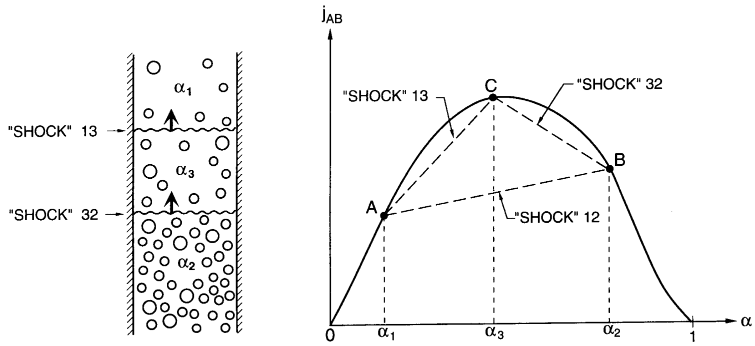

Figure 2: Shock instability for $\alpha_1 < \alpha_2$.