Cavity Detachment Models

The other regions of the flow that require careful consideration are the points at which the free streamlines "detach" from the body. We use the word "detachment" to avoid confusion with the process of separation of the boundary layer. Thus the words "separation point" are reserved for boundary layer separation.

Since most of the mathematical models assume incompressible and irrotational potential flow, it is necessary to examine the prevailing conditions at a point at which a streamline in such a flow detaches from a solid surface. We first observe that if the pressure in the cavity is assumed to be lower than at any other point in the liquid, then the free surface must be convex viewed from the liquid. This precludes free streamlines with negative curvatures (the sign is taken to be positive for a convex surface). Second, we distinguish between the two geometric circumstances shown in Figure 1. Abrupt detachment is the term applied to

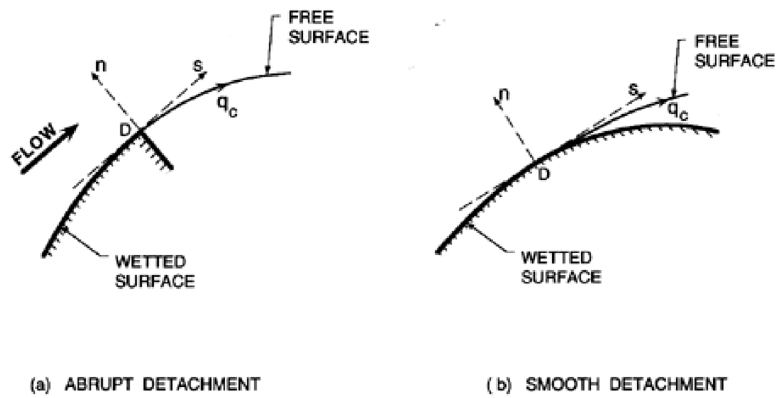

Figure 1: Notation used in the discussion of the detachment of a free streamline from a solid body.

the case in which the free surface leaves the solid body at a vertex or discontinuity in the slope of the body surface.

For convenience in the discussion we define a coordinate system, (s, n), whose origin is at the detachment point or vertex. The direction of the coordinate, s, coincides with the direction of the velocity vector at the detachment point and the coordinate, n, is perpendicular to the solid surface. It is sufficiently accurate for present purposes to consider the flow to be locally planar and to examine the nature of the potential flow solutions in the immediate neighborhood of the detachment point, D. Specifically, it is important to identify the singular behavior at D. This is most readily accomplished by using polar coordinates, (r, ϑ), where $z = s + in = r\, e^{i\vartheta}$, and by considering the expansion of the logarithmic hodograph variable, ϖ (see section (Nua)), as a power series in z. Since, to first order, $Re\{\varpi\} = 0$ on $\vartheta = 0$ and $Im\{\varpi\} = 0$ on $\vartheta = \pi$, it follows that, in general, the first term in this expansion is

$$\varpi = -Ciz^{\frac{1}{2}} + \cdots \qquad \text{(Nuc1)}$$

where the real constant C would be obtained as a part of the solution to the specific flow. From equation (Nuc1), it follows that

$$w = q_c\{1 + Ciz^{\frac{1}{2}} + \cdots\} \qquad \text{(Nuc2)}$$

$$f = \phi + i\psi = q_c\left\{z + \frac{2}{3}Ciz^{\frac{3}{2}} + \cdots\right\} \qquad \text{(Nuc3)}$$

and the following properties of the flow at an abrupt detachment point then become evident. First, from equation (Nuc2) it is clear that the acceleration of the fluid tends to infinity as one approaches the detachment point along the wetted surface. This, in turn, implies an infinite, favorable pressure gradient. Moreover, in order for the wetted surface velocity to be lower than that on the free surface (and therefore for the wetted surface pressure to be higher than that in the cavity), it is necessary for C to be a *positive* constant. Second, since the shape of the free surface, $\psi = 0$, is given by

$$y + Re\left\{\frac{2}{3}Cz^{\frac{3}{2}} + \cdots\right\} = 0 \tag{Nuc4}$$

it follows that the curvature of that surface becomes infinite as the detachment point is approached along the free surface. The sign of C also implies that the free surface is convex viewed from within the liquid. The modifications to these characteristics as a result of a boundary layer in a real flow were studied by Ackerberg (1970); it seems that the net effect of the boundary layer on abrupt detachment is not very significant. We shall delay further discussion of the practical implications of these analytical results until later.

Turning attention to the other possibility sketched in Figure 1, "smooth detachment," one must first ask why it should be any different from abrupt detachment. The reason is apparent from one of the results of the preceding paragraph. An infinite, convex free-surface curvature at the detachment point is geometrically impossible at a smooth detachment point because the free surface would then cut into the solid surface. However, the position of the smooth detachment point is initially unknown. One can therefore consider a whole family of solutions to the particular flow, each with a different detachment point. There may be one such solution for which the strength of the singularity, C, is identically zero, and this solution, unlike all the others, is viable since its free surface does not cut into the solid surface. Thus the condition that the strength of the singularity, C, be zero determines the location of the smooth detachment point. These circumstances and this condition were first recognized independently by Brillouin (1911) and by Villat (1914), and the condition has become known as the Brillouin-Villat condition. Though normally applied in planar flow problems, it has also been used by Armstrong (1953), Armstrong and Tadman (1954), and Brennen (1969a) in axisymmetric flows.

The singular behavior at a smooth detachment point can be examined in a manner similar to the above analysis of an abrupt detachment point. Since the one-half power in the power law expansion of ϖ is now excluded, it follows from the conditions on the free and wetted surfaces that

$$\varpi = -Ciz^{\frac{3}{2}} + \cdots \tag{Nuc5}$$

where C is a different real constant, the strength of the three-half power singularity. By parallel evaluation of w and f one can determine the following properties of the flow at a smooth detachment point. The velocity and pressure gradients approach zero (rather than infinity) as the detachment point is approached along the wetted surface. Also, the curvature of the free surface approaches that of the solid surface as the detachment point is approached along the free surface. Thus the name "smooth detachment" seems appropriate.

Having established these models for the detachment of the free streamlines in potential flow, it is important to emphasize that they are models and that viscous boundary-layer and surface-energy effects (surface tension and contact angle) that are omitted from the above discussions will, in reality, have a substantial influence in determining the location of the actual detachment points. This can be illustrated by comparing the locations of smooth detachment from a cavitating sphere with experimentally measured locations. As can readily be seen from Figures 2 and 3, the predicted detachment locations are substantially upstream of the actual detachment points. Moreover, the experimental data exhibit some systematic variations

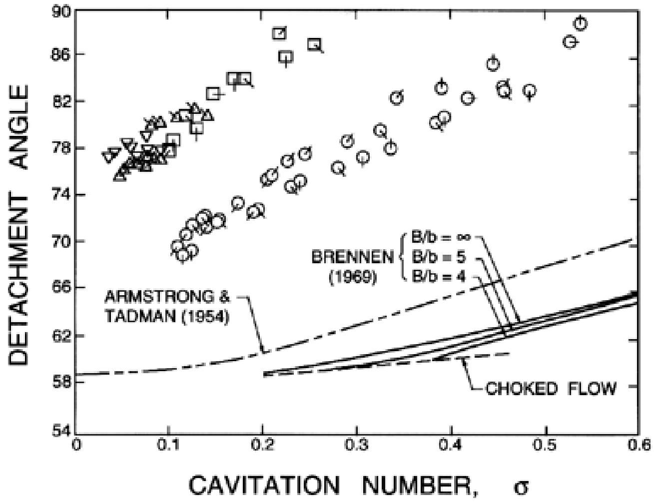

Figure 2: Observed and calculated locations of free surface detachment for a cavitating sphere. The detachment angle is measured from the front stagnation point. The analytical results using the smooth detachment condition are from Armstrong and Tadman (1954) and Brennen (1969a), in the latter case for different water tunnel to sphere radius ratios, B/b (see section (Nuf)). The experimental results are for different sphere diameters as follows: 7.62 cm (⊙) and 2.86 cm (□) from Brennen (1969a), 5.08 cm (△) and 3.81 cm (▽) from Hsu and Perry (1954). Tunnel velocities are indicated by the additional ticks at cardinal points as follows: 4.9 m/s (NW), 6.1 m/s (N), 7.6 m/s (NE), 9.1 m/s (E), 10.7 m/s (SE), 12.2 m/s (S) and 13.7 m/s (SW).

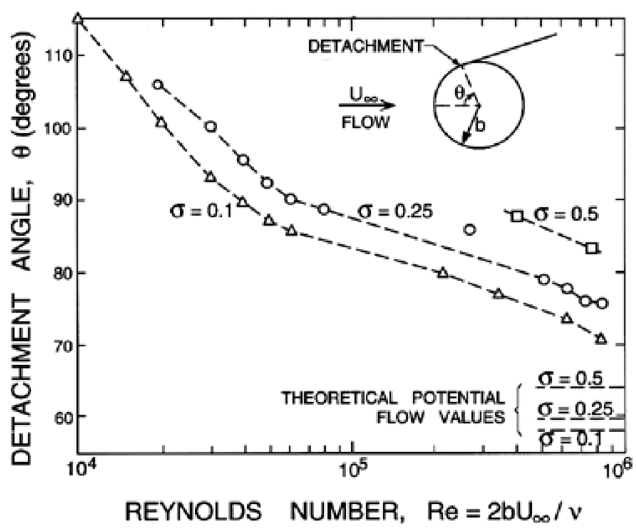

Figure 3: Observed free surface detachment points from spheres for various cavitation numbers, σ, and Reynolds numbers. Also shown are the potential flow values using the smooth detachment condition. Adapted from Brennen (1969b).

with the size of the sphere and the tunnel velocity. Exploring these scaling effects, Brennen (1969b) interpolated between the data to construct the variations with Reynolds number shown in Figure 3. This data clearly indicates that the detachment locations are determined primarily by viscous, boundary-layer effects. However, one must add that all of the experimental data used for Figure 3 was for metal spheres and that surface-energy effects and, in particular, contact-angle effects probably also play an important role (see Ackerberg 1975). The effect of the surface tension of the liquid seems to be relatively minor (Brennen 1970).

It is worth noting that, despite the discrepancies between the observed locations of detachment and those

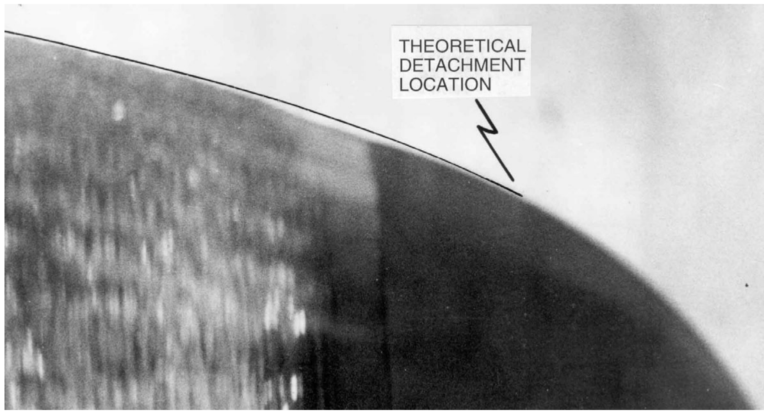

Figure 4: Comparison of the theoretical and experimental profiles of a fully developed cavity behind a sphere. The flow is from the right to the left. From Brennen (1969a).

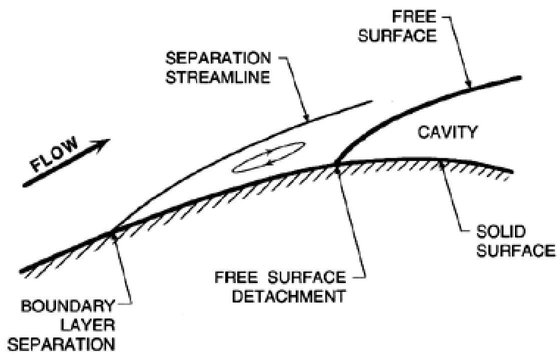

Figure 5: Model of the flow in the vicinity of a smooth detachment point. Adapted from Arakeri (1975).

predicted by the smooth detachment condition, the profile of the cavity is not as radically affected as one might imagine. Figure 4, taken from Brennen (1969a), is a photograph showing the profile of a fully developed cavity on a sphere. On it is superimposed the profile of the theoretical solution. Note the close proximity of the profiles despite the substantial discrepancy in the detachment points.

The viscous flow in the vicinity of an actual smooth detachment point is complex and still remains to be completely understood. Arakeri (1975) examined this issue experimentally using Schlieren photography to determine the behavior of the boundary layer and observed that boundary layer separation occurred upstream of free surface detachment as sketched in Figure 5 and shown in Figure 6. Arakeri also generated a quasi-empirical approach to the prediction of the distance between the separation and detachment locations, and this model seemed to produce detachment positions that were in good agreement with the observations. Franc and Michel (1985) studied this same issue both analytically and through experiments on hydrofoils, and their criterion for the detachment location has been used by several subsequent investigators.

In practice many of the methods used to solve free streamline problems involving detachment from a smooth surface simply assume a known location of detachment based on experimental observations (for example, Furuya and Acosta 1973) and neglect the difficulties associated with the resulting abrupt detachment solution.

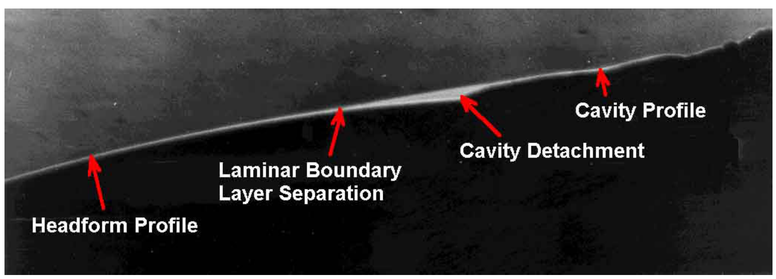

Figure 6: Schlieren photograph showing boundary layer separation upstream of the free surface detachment on an axisymmetric headform. The cavitation number is 0.39 and the tunnel velocity is 8.1 m/s. The actual distance between the separation and detachment points is about 0.28 cm. Reproduced from Arakeri (1975) with permission of the author.