# Disperse Phase Number Continuity

Complementary to the equations of conservation of mass are the equations governing the conservation of the number of bubbles, drops, particles, etc. that constitute a disperse phase. If no such *particles* are created or destroyed within the elemental volume and if the number of *particles* of the disperse component, $D$, per unit total volume is denoted by $n_D$, it follows that

$$\frac{\partial n_D}{\partial t} + \frac{\partial}{\partial x_i}(n_D u_{Di}) = 0 \tag{Nbc1}$$

This will be referred to as the Disperse Phase Number Equation (DPNE).

If the volume of the particles of component $D$ is denoted by $v_D$ it follows that

$$\alpha_D = n_D v_D \tag{Nbc2}$$

and substituting this into equation (Nbc1) one obtains

$$\frac{\partial}{\partial t}(n_D \rho_D v_D) + \frac{\partial}{\partial x_i}(n_D u_{Di} \rho_D v_D) = \mathcal{I}_D \tag{Nbc3}$$

Expanding this equation using equation (Nbc1) leads to the following relation for $\mathcal{I}_D$:

$$\mathcal{I}_D = n_D \left( \frac{\partial(\rho_D v_D)}{\partial t} + u_{Di} \frac{\partial(\rho_D v_D)}{\partial x_i} \right) = n_D \frac{D_D}{D_D t}(\rho_D v_D) \tag{Nbc4}$$

where $D_D/D_D t$ denotes the Lagrangian derivative following the disperse phase. This demonstrates a result that could, admittedly, be assumed, a priori. Namely that the rate of transfer of mass to the component $D$ in each particle, $\mathcal{I}_D/n_D$, is equal to the Lagrangian rate of increase of mass, $\rho_D v_D$, of each particle.

It is sometimes convenient in the study of bubbly flows to write the bubble number conservation equation in terms of a population, $\eta$, of bubbles per unit *liquid* volume rather than the number per unit total volume, $n_D$. Note that if the bubble volume is $v$ and the volume fraction is $\alpha$ then

$$\eta = \frac{n_D}{(1-\alpha)} \; ; \; n_D = \frac{\eta}{(1+\eta v)} \; ; \; \alpha = \eta \frac{v}{(1+\eta v)} \tag{Nbc5}$$

and the bubble number conservation equation can be written as

$$\frac{\partial u_{Di}}{\partial x_i} = -\frac{(1+\eta v)}{\eta} \frac{D_D}{D_D t} \left( \frac{\eta}{1+\eta v} \right) \tag{Nbc6}$$

If the number population, $\eta$, is assumed uniform and constant (which requires neglect of slip and the assumption of liquid incompressibility) then equation (Nbc6) can be written as

$$\frac{\partial u_{Di}}{\partial x_i} = \frac{\eta}{1+\eta v} \frac{D_D v}{D_D t} \tag{Nbc7}$$

In other words the divergence of the velocity field is directly related to the Lagrangian rate of change in the volume of the bubbles.