Heat Transfer between Separated Phases

In the preceding section, the rate of heat transfer, $\mathcal{QI}_N$, to each phase, N, from the other phases was left undefined. Now we address the functional form of this rate of heat transfer in the illustrative case of a two-phase flow consisting of a disperse solid particle or liquid droplet phase and a gaseous continuous phase.

In section (Nbg), we defined a relaxation time that typifies the natural attenuation of velocity differences between the phases. In an analogous manner, the temperatures of the phases might be different down-stream of a flow disturbance and consequently there would be a second *relaxation* time associated with the equilibration of temperatures through the process of heat transfer between the phases. This temperature relaxation time is denoted by t_T and can be obtained by equating the rate of heat transfer from the continuous phase to the particle with the rate of increase of heat stored in the particle. The heat transfer to the particle can occur as a result of conduction, convection or radiation and there are practical flows in which each of these mechanisms are important. For simplicity, we shall neglect the radiation component. Then, if the relative motion between the particle and the gas is sufficiently small, the only contributing mechanism is conduction and it will be limited by the thermal conductivity, k_C, of the gas (since the thermal conductivity of the particle is usually much greater). Then the rate of heat transfer to a particle (radius R) will be given approximately by $2\pi R k_C (T_C - T_D)$ where T_C and T_D are representative temperatures of the gas and particle respectively.

Now we add in the component of heat transfer by the convection caused by relative motion. To do so we define the Nusselt number, Nu, as twice the ratio of the rate of heat transfer with convection to that without convection. Then the rate of heat transfer becomes Nu times the above result for conduction. Typically, the Nusselt number is a function of both the Reynolds number of the relative motion, $Re = 2WR/\nu_C$ (where W is the typical magnitude of $(u_{Di} - u_{Ci})$), and the Prandtl number, $Pr = \rho_C \nu_C c_{pC}/k_C$. One frequently used expression for Nu (see Ranz and Marshall 1952) is

$$Nu = 2 + 0.6 Re^{\frac{1}{2}} Pr^{\frac{1}{3}} \tag{Nbi1}$$

and, of course, this reduces to the pure conduction result, $Nu = 2$, when the second term on the right hand side is small.

Assuming that the particle temperature has a roughly uniform value of T_D, it follows that

$$\mathcal{QI}_D = 2\pi R k_C Nu (T_C - T_D) n_D = \rho_D \alpha_D c_{sD} \frac{DT_D}{Dt} \tag{Nbi2}$$

where the material derivative, D/Dt, follows the particle. This provides the equation that must be solved for T_D namely

$$\frac{DT_D}{Dt} = \frac{Nu}{2} \frac{(T_C - T_D)}{t_T} \tag{Nbi3}$$

where

$$t_T = c_{sD} \rho_D R^2 / 3k_C \tag{Nbi4}$$

Clearly t_T represents a typical time for equilibration of the temperatures in the two phases, and is referred to as the *temperature relaxation time*.

The above construction of the temperature relaxation time and the equation for the particle temperature represents perhaps the simplest formulation that retains the essential ingredients. Many other effects may

become important and require modification of the equations. Examples are the rarefied gas effects and turbulence effects. Moreover, the above was based on a uniform particle temperature and steady state heat transfer correlations; in many flows heat transfer to the particles is highly transient and a more accurate heat transfer model is required. For a discussion of these effects the reader is referred to Rudinger (1969) and Crowe *et al.* (1998).