

Single Droplet Mechanics

The combustion of liquid fuels in droplet form or of solid fuels in particulate form constitute a very important component of our industrialized society. Spray evaporation is important, in part because it constitutes the first stage in the combustion of atomized liquid fuels in devices such as industrial furnaces, diesel engines, liquid rocket engines or gas turbines. Consequently the mechanics of the evaporation and subsequent combustion have been extensively documented and studied (see, for example, Williams 1965, Glassman 1977, Law 1982, Faeth 1983, Kuo 1986) and their air pollution consequences examined in detail (see, for example, Flagan and Seinfeld 1988). It is impossible to present a full review of these subjects within the confines of this book, but it is important and appropriate to briefly review some of the basic multiphase flow phenomena that are central to these processes.

An appropriate place to start is with evaporation of a single droplet in a quiescent environment and we will follow the description given in Flagan and Seinfeld (1988). Heat diffusing inward from the combustion zone, either one surrounding a gas/droplet cloud or one located around an individual droplet, will cause the heating and evaporation of the droplet(s). It transpires that it is adequate for most purposes to model single droplet evaporation as a steady state process (assuming the droplet radius is only varying slowly). Since the liquid density is much greater than the vapor density, the droplet radius, R , can be assumed constant in the short term and this permits a steady flow analysis in the surrounding gas. Then, since the outward flow of total mass and of vapor mass at every radius, r , is equal to \dot{m}_V and there is no net flux of the other gas, conservation of total mass and conservation of vapor lead through equations (Nbb2) and (Nbb10) and Fick's Law (Nbd1) to

$$\frac{\dot{m}_V}{4\pi} = \rho u r^2 = \rho(u)_{r=R} R^2 \quad (\text{Nog1})$$

and

$$\frac{\dot{m}_V}{4\pi} = \rho u r^2 x_V - \rho r^2 D \frac{dx_V}{dr} \quad (\text{Nog2})$$

where D is the mass diffusivity. These represent equations to be solved for the mass fraction of the vapor, x_V . Eliminating u and integrating produces

$$\frac{\dot{m}_V}{4\pi} = \rho R D \ln \left(1 + \frac{(x_V)_{r=\infty} - (x_V)_{r=R}}{(x_V)_{r=R} - 1} \right) \quad (\text{Nog3})$$

Next we examine the heat transfer in this process. The equation governing the radial convection and diffusion of heat is

$$\rho u c_p \frac{dT}{dr} = \frac{1}{r^2} \frac{d}{dr} \left(r^2 k \frac{dT}{dr} \right) \quad (\text{Nog4})$$

where c_p and k are representative averages of, respectively, the specific heat at constant pressure and the thermal conductivity of the gas. Substituting for u from equation (Nog1) this can be integrated to yield

$$\dot{m}_V c_p (T + C) = 4\pi r^2 k \frac{dT}{dr} \quad (\text{Nog5})$$

where C is an integration constant that is evaluated by means of the boundary condition at the droplet surface. The heat required to vaporize a unit mass of fuel whose initial temperature is denoted by T_i is

clearly that required to heat it to the saturation temperature, T_e , plus the latent heat, \mathcal{L} , or $c_s(T_e - T_i) + \mathcal{L}$. The second contribution is usually dominant so the heat flux at the droplet surface can be set as:

$$4\pi R^2 k \left(\frac{dT}{dr} \right)_{r=R} = \dot{m}_V \mathcal{L} \quad (\text{Nog6})$$

Using this boundary condition, C can be evaluated and equation (Nog5) further integrated to obtain

$$\frac{\dot{m}_V}{4\pi} = \frac{Rk}{c_p} \ln \left\{ 1 + \frac{c_p(T_{r=\infty} - T_{r=R})}{\mathcal{L}} \right\} \quad (\text{Nog7})$$

To solve for $T_{r=R}$ and $(x_V)_{r=R}$ we eliminate \dot{m}_V from equations (Nog3) and (Nog7) and obtain

$$\frac{\rho D c_p}{k} \ln \left(1 + \frac{(x_V)_{r=\infty} - (x_V)_{r=R}}{(x_V)_{r=R} - 1} \right) = \ln \left(1 + \frac{c_p(T_{r=\infty} - T_{r=R})}{\mathcal{L}} \right) \quad (\text{Nog8})$$

Given the transport and thermodynamic properties k , c_p , \mathcal{L} , and D (neglecting variations of these with temperature) as well as $T_{r=\infty}$ and ρ , this equation relates the droplet surface mass fraction, $(x_V)_{r=R}$, and temperature $T_{r=R}$. Of course, these two quantities are also connected by the thermodynamic relation

$$(x_V)_{r=R} = \frac{(\rho_V)_{r=R}}{\rho} = \frac{(p_V)_{r=R}}{p} \frac{\mathcal{M}_V}{\mathcal{M}} \quad (\text{Nog9})$$

where \mathcal{M}_V and \mathcal{M} are the molecular weights of the vapor and the mixture. Equation (Nog8) can then be solved given the relation (Nog9) and the saturated vapor pressure p_V as a function of temperature. Note that since the droplet size does not occur in equation (Nog8), the surface temperature is independent of the droplet size.

Once the surface temperature and mass fraction are known, the rate of evaporation can be calculated from equation (Nog4) by substituting $\dot{m}_V = 4\pi\rho_L R^2 dR/dt$ and integrating to obtain

$$R^2 - (R_{t=0})^2 = \left\{ \frac{2k}{c_p} \ln \left(1 + \frac{c_p(T_{r=\infty} - T_{r=R})}{\mathcal{L}} \right) \right\} t \quad (\text{Nog10})$$

Thus the time required for complete evaporation, t_{ev} , is

$$t_{ev} = c_p R_{t=0}^2 \left\{ 2k \ln \left(1 + \frac{c_p(T_{r=\infty} - T_{r=R})}{\mathcal{L}} \right) \right\}^{-1} \quad (\text{Nog11})$$

This quantity is important in combustion systems. If it approaches the residence time in the combustor this may lead to incomplete combustion, a failure that is usually avoided by using atomizing nozzles that make the initial droplet size, $R_{t=0}$, as small as possible.

Having outlined the form of the solution for an evaporating droplet, albeit in the simplest case, we now proceed to consider the combustion of a single droplet.