Introduction to Single Particle Motion

This section will briefly review the issues and problems involved in constructing the equations of motion for individual particles, drops or bubbles moving through a fluid. For convenience we shall use the generic name *particle* to refer to the finite pieces of the disperse phase or component. The analyses are implicitly confined to those circumstances in which the interactions between neighboring particles are negligible. In very dilute multiphase flows in which the particles are very small compared with the global dimensions of the flow and are very far apart compared with the particle size, it is often sufficient to solve for the velocity and pressure, $u_i(x_i, t)$ and $p(x_i, t)$, of the continuous suspending fluid while ignoring the particles or disperse phase. Given this solution one could then solve an equation of motion for the particle to determine its trajectory. This chapter will focus on the construction of such a particle or bubble equation of motion.

The body of fluid mechanical literature on the subject of flows around particles or bodies is very large indeed. Here we present a summary that focuses on a spherical particle of radius, R, and employs the following common notation. The components of the translational velocity of the center of the particle will be denoted by $V_i(t)$. The velocity that the fluid would have had at the location of the particle center in the absence of the particle will be denoted by $U_i(t)$. Note that such a concept is difficult to extend to the case of interactive multiphase flows. Finally, the velocity of the particle relative to the fluid is denoted by $W_i(t) = V_i - U_i$.

Frequently the approach used to construct equations for $V_i(t)$ (or $W_i(t)$) given $U_i(x_i, t)$ is to individually estimate all the fluid forces acting on the particle and to equate the total fluid force, F_i, to $m_p dV_i/dt$ (where m_p is the particle mass, assumed constant). These fluid forces may include forces due to buoyancy, added mass, drag, etc. In the absence of fluid acceleration ($dU_i/dt = 0$) such an approach can be made unambiguously; however, in the presence of fluid acceleration, this kind of heuristic approach can be misleading. Hence we concentrate in the next few sections on a fundamental fluid mechanical approach, that minimizes possible ambiguities. The classical results for a spherical particle or bubble are reviewed first. The analysis is confined to a suspending fluid that is incompressible and Newtonian so that the basic equations to be solved are the continuity equation

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{Nea1}$$

and the Navier-Stokes equations

$$\rho_C \left\{ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right\} = -\frac{\partial p}{\partial x_i} + \rho_C \nu_C \frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{Nea2}$$

where ρ_C and ν_C are the density and kinematic viscosity of the suspending fluid. It is assumed that the only external force is that due to gravity, g. Then the actual pressure is $p' = p - \rho_C g z$ where z is a coordinate measured vertically upward.

Furthermore, in order to maintain clarity we confine our attention to rectilinear relative motion in a direction conveniently chosen to be the x_1 direction.