Particle Interaction Models

It is self-evident that the rheology of granular flows will be strongly influenced by the dynamics of particle-particle interactions. Consequently the solid mechanics and dynamics of those interactions must be established prior to a discussion of the rheology of the overall flow. We note that the relation between the rheology and the particle-particle interaction can quite subtle (Campbell 2002, 2003).

Early work on rapid granular material flows often assumed instantaneous, binary collisions between particles, in other words a *hard particle model* (see, for example, Campbell and Brennen 1985a, b). While this assumption may be valid in some applications, it is now recognized that the high shear rates required to achieve such flow conditions are unusual (Campbell 2002) and that most practical granular flows have more complex particle-particle interactions that, in turn, lead to more complex rheologies. To illustrate this we will confine the discussion to the particular form of particle-particle interaction most often used in computer simulations. We refer to the model of the particle-particle dynamics known as the *soft particle model*, depicted in figure 1. First utilized by Cundall and Strack (1979), this admittedly simplistic model consists of a spring, K_n, and dashpot, C, governing the normal motion and a spring, K_s, and Coulomb friction coefficient, μ^*, governing the tangential motion during the contact and deformation of two particles of mass, m_p. The model has been subject to much study and comparison with experiments, for example by Bathurst and Rothenburg (1988). Though different normal and tangential spring constants are often used we will, for simplicity, characterize them using a single spring constant (Bathurst and Rothenburg show that K_s/K_n determines the bulk Poisson's ratio) that, neglecting the effects of non-linear Hertzian-like deformations will be characterized by a simple linear elastic spring constant, K. Note that as described by Bathurst and Rothenburg, the Young's modulus of the bulk material will be proportional to K. Note also that K will be a function not only of properties of the solid material but also of the geometry of the contact points. Furthermore, it is clear that the dashpot constant, C, will determine the loss of energy during normal collisions and will therefore be directly related to the coefficient of restitution for normal collisions. Consequently, appropriate values of C can be determined from known or measured coefficients

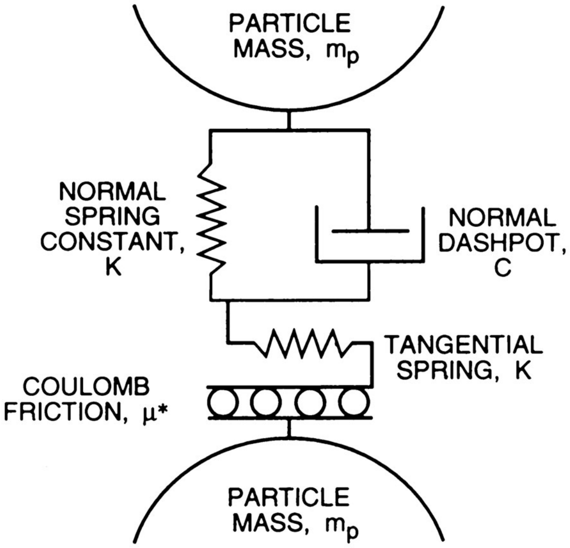

Figure 1: Schematic of the soft particle model of particle interaction.

of restitution, ϵ; the specific relation is

$$\epsilon = exp\left(-\pi C/\left[2m_pK - C^2\right]^{\frac{1}{2}}\right) \tag{Npb1}$$

Note that this particle interaction model leads to a collision time for individual binary collisions, t_c, that is the same for all collisions and is given by

$$t_c = \pi m_p/\left[2m_pK - C^2\right]^{\frac{1}{2}} \tag{Npb2}$$

Before leaving the subject of individual particle interactions, several cautionary remarks are appropriate. Models such as that described above and those used in most granular flow simulations are highly simplified and there are many complications whose effects on the granular flow rheology remain to be explored. For example, the spring stiffnesses and the coefficients of restitution are often far from constant and depend on the geometry of the particle-particle contacts and velocity of the impact as well as other factors such as the surface roughness. The contact stiffnesses may be quite non-linear though Hertzian springs (in which the force is proportional to the displacement raised to the 3/2 power) can be readily incorporated into the computer simulations. We also note that velocities greater than a few cm/s will normally lead to plastic deformation of the solid at the contact point and to coefficients of restitution that decrease with increasing velocity (Goldsmith 1960, Lun and Savage 1986). Boundary conditions may also involve complications since the coefficient of restitution of particle-wall collisions can depend on the wall thickness in a complicated way (Sondergard *et al.* 1989). Appropriate tangential coefficients are even more difficult to establish. The tangential spring stiffness may be different from the normal stiffness and may depend on whether or not slippage occurs during contact. This introduces the complications of tangential collisions studied by Maw *et al.*(1976, 1981), Foerster *et al.*(1994) and others. The interstitial fluid can have a major effect on the interaction dynamics; further comment on this is delayed until section (Npn). The point to emphasize here is that much remains to be done before all the possible effects on the granular flow rheology have been explored.