Mohr-Coulomb Models

As a specific example, the Mohr-Coulomb-Jenike-Shield model (Jenike and Shield 1959) utilizes a Mohr's circle diagram to define a yield criterion and it is assumed that once the material starts to flow, the stresses must continue to obey that yield criterion. For example, in the flow of a cohesionless material, one might utilize a Coulomb friction yield criterion in which it is assumed that the ratio of the principal shear stress to the principal normal stress is simply given by the *internal friction angle*, ϕ, that is considered to be a material property. In a two-dimensional flow, for example, this would imply the following relation between the stress tensor components:

$$\left\{ \left(\frac{\sigma_{xx} - \sigma_{yy}}{2} \right)^2 + \sigma_{xy}^2 \right\}^{\frac{1}{2}} = -sin\phi \left(\frac{\sigma_{xx} + \sigma_{yy}}{2} \right) \tag{Nph1}$$

where the left hand side would be less than the right in regions where the material is not flowing or deforming.

However, equations (Npg1), (Npg2) and (Nph1) are insufficient and must be supplemented by at least two further relations. In the Mohr-Coulomb-Jenike-Shield model, an assumption of isotropy is also made; this assumes that the directions of principal stress and principal strain rate correspond. For example, in two-dimensional flow, this implies that

$$\frac{\sigma_{xx} - \sigma_{yy}}{\sigma_{xy}} = \frac{2\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)}{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}} \tag{Nph2}$$

It should be noted that this part of the model is particularly suspect since experiments have shown substantial departures from isotropy. Finally one must also stipulate some relation for the solids fraction α and typically this has been considered a constant equal to the critical solids fraction or to the maximum shearable solids fraction. This feature is also very questionable since even slow flows such as occur in hoppers display substantial decreases in α in the regions of faster flow.