Effect of Interstitial Fluid

All of the preceeding discussion assumed that the effect of the interstitial fluid was negligible. When the fluid dynamics of the interstitial fluid has a significant effect on a granular flow, analysis of the rheology becomes even more complex and our current understanding is quite incomplete. It was Bagnold (1954) who first attempted to define those circumstances in which the interstitial fluid would begin to effect the rheology of a granular flow. Bagnold introduced a parameter that included the following dimensionless quantity

$$Ba = \rho_S D^2 \dot{\gamma}/\mu_L \quad \text{(Npn1)}$$

where $\dot{\gamma}$ is the shear rate; we will refer to Ba as the Bagnold number. It is simply a measure of the stresses communicated by particle-particle collisions (given according to kinetic theory ideas by $\rho_S V^2$ where V is the typical random velocity of the particles that, in turn, is estimated to be given by $V = D\dot{\gamma}$) to the viscous stress in the fluid, $\mu_L \dot{\gamma}$. On the basis of his experimental observations Bagnold concluded that when the value of Ba was less than about 40, the viscous fluid stresses dominate and the mixture exhibits a Newtonian rheology in which the shear stress and the strain rate ($\dot{\gamma}$) are linearly related; he called this the viscous regime. On the other hand when Ba is greater than about 400, the direct particle-particle (and particle-wall) interactions dominate and the stresses become proportional to the square of the strain rate. The viscous regime can be considered the dense suspension regime and many other sections of this book are relevant to those circumstances in which the direct particle-particle and particle-wall interactions play a minor role in the mixture rheology. In this section we focus attention on the other limit in which the effect of the interstitial fluid is small and the rheology is determined by the direct interactions of the particles with themselves and with the walls.

Further documentation of the transitional Bagnold number emerged from the experiments of Zeininger and Brennen (1985) who measured the discharge flow rates from conical hoppers for a number of different granular materials and compared the flow rates in air with those from underwater experiments. To provide a valid comparison it is necesssary to account for the difference in the effective material densities in the air and water experiments by defining a dimensionless flow rate as

$$u_d = U_d / \left[gD(1 - \rho_L/\rho_S) \right]^{1/2} \quad \text{(Npn2)}$$

where U_d is the material velocity at the discharge, D is the discharge diameter, ρ_L and ρ_S are the fluid and solid densities and g is the acceleration due to gravity. This dimensionless discharge is consistent with that which emerges from all of the theoretical analyses of granular material flows in hoppers (see, for example, Brennen and Pearce (1978) and Nguyen *et al.* (1979)). The factor $(1 - \rho_L/\rho_S)$ accounts for the effective gravitational acceleration in the underwater hopper experiments and is clearly close to unity for the flows in air.

Figure 1 presents the Zeininger and Brennen data on the dimensionless flow rate, u_d, as a function of an appropriate Bagnold number, Ba. In these hopper flows the appropriate velocity gradient, $\dot{\gamma}$, to use in the Bagnold number is the extensional deformation rate that dominates hopper flows. In the interior of the hopper near the discharge the velocity, U, will vary according to $U_d(r_d/r)^{1/2}$ (where r is the radius from the geometrical center of the cone and $r = r_d$ at the discharge). Thus it follows that the appropriate $\dot{\gamma}$ is

$$\dot{\gamma} = -\left[\frac{dU}{dr} \right]_{r=r_d} = 4U_d \sin\theta / D \quad \text{(Npn3)}$$

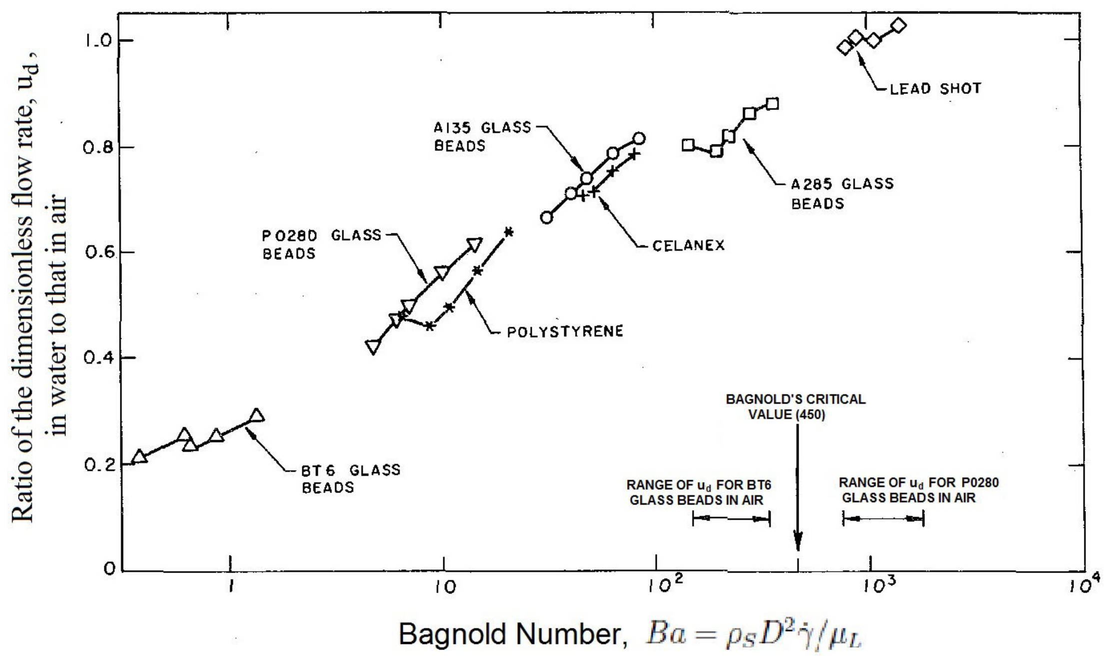

Figure 1: The ratio of the non=dimensional flow rate, u_d, from hoppers in water to that in air as a function of the Bagnold number, $Ba = \rho_S D^2 \dot{\gamma}/\mu_L$, for a variety of granular materials, BT6 glass beads ($D = 0.26mm$), P0280 glass beads ($D = 0.61mm$), A135 glass beads ($D = 1.39mm$), A285 glass beads ($D = 2.92mm$). lead shot ($D = 2.26mm$), polystyrene beads ($D \approx 2.72mm$) and Celanex beads ($D \approx 3.25mm$) and for a series of conical hoppers with half-angles ranging from $10°$ to $35°$ and discharge diameters of about $3cm$. Adapted from Zeininger and Brennen (1985).

where θ is the half-angle of the conical hopper. This is the shear rate used in the Bagnold number of Figure (Npn1). It is remarkable that all of the data closely defines a single curve that asymptotes to unity at a valne of Ba of the order of 500. In other words a value essentially the same as that noted by Bagnold (i.e. 450). For each material, data for each of the hoppers is shown connected by a line and it would appear that the variation from hopper to hopper is consistent with the variation from material to material. We note that the ranges of Bagnold numbers for the air experiments with the smaller glass beads are also indicated in Figure (Npn1) and suggest that, in those experiments there is some small interstitial effect of the air.