Wall Effects and Choked Flows

Several useful results follow from the application of basic fluid mechanical principles to cavity flows constrained by uniform containing walls. Such would be the case, for example, for experiments in water tunnels. Consequently, in this section, we focus attention on the issue of wall effects in cavity flows and on the related phenomenon of *choked flow*. Anticipating some of the results of section (Nuf), we observe that, for the same cavitation number, the narrower the tunnel relative to the body, the broader and longer the cavity becomes and the lower the drag coefficient. For a finite tunnel width, there is a critical cavitation number, σ_c, at which the cavity becomes infinitely long and no solutions exist for $\sigma < \sigma_c$. The flow is said to be *choked* at this limiting condition because, for a fixed tunnel pressure and a fixed cavity pressure, a minimum cavitation number implies an upper limit to the tunnel velocity. Consequently the choking phenomenon is analogous to that which occurs in a the nozzle flow of a compressible fluid. The phenomenon is familiar to those who have conducted experiments on fully developed cavity flows in water tunnels. When one tries to exceed the maximum, choked velocity, the water tunnel pressure rises so that the cavitation number remains at or above the choked value.

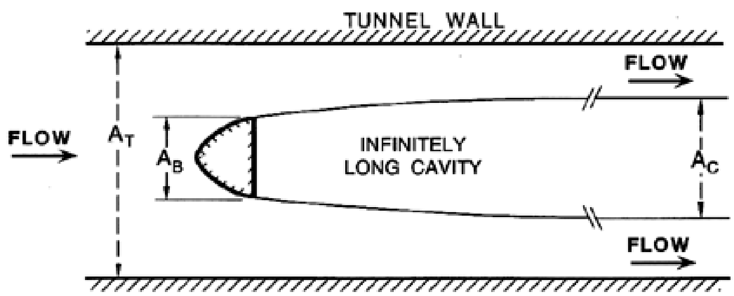

Figure 1: Body with infinitely long cavity under choked flow conditions.

In the choked flow limit of an infinitely long cavity, application of the equations of conservation of mass, momentum, and energy lead to some simple relationships for the parameters of the flow. Referring to Figure 1, consider a body with a frontal projected area of A_B in a water tunnel of cross-sectional area, A_T. In the limit of an infinitely long cavity, the flow far downstream will be that of a uniform stream in a straight annulus, and therefore conservation of mass requires that the limiting cross-sectional area of the cavity, A_C, be given by

$$\frac{A_C}{A_T} = 1 - \frac{U_\infty}{q_c} = 1 - (1+\sigma_c)^{-\frac{1}{2}} \tag{Nud1}$$

which leads to

$$\sigma_c \approx 2A_C/A_T \tag{Nud2}$$

for the small values of the area ratio that would normally apply in water tunnel tests. The limiting cavity cross-sectional area, A_C, will be larger than the frontal body area, A_B. However, if the body is streamlined these areas will not differ greatly and therefore, according to equation (Nud1), a first approximation to the value of σ_c would be

$$\sigma_c \approx 1 - \left(1 - \frac{A_B}{A_T}\right)^2 \tag{Nud3}$$

Note, as could be anticipated, that the larger the blockage ratio, A_B/A_T, the higher the choked cavitation number, σ_c. Note, also, that the above equations assume frictionless flow since the relation, $q_c/U_\infty =$

$(1+\sigma)^{\frac{1}{2}}$, was used. Hydraulic losses along the length of the water tunnel would introduce other effects in which choking would occur at the end of the tunnel working section in a manner analogous to the effects of friction in compressible pipe flow.

A second, useful result emerges when the momentum theorem is applied to the flow, again assumed frictionless. Then, in the limit of choked flow, the drag coefficient, $C_D(\sigma_c)$, is given by

$$C_D(\sigma_c) = \frac{A_T}{A_B}\left[\sigma_c - 2\left\{(1+\sigma_c)^{\frac{1}{2}} - 1\right\}\right] \tag{Nud4}$$

When $\sigma_c \ll 1$ it follows from equations (Nud2) and (Nud4) that

$$C_D(\sigma_c) \approx \frac{A_C^2}{A_B A_T} \approx \frac{A_C}{A_B}\frac{\sigma_c}{2} \tag{Nud5}$$

where, of course, A_C/A_B, would depend on the shape of the body. The approximate validity of this result can be observed in section (Nuf); it is clear that for the 30° half-angle wedge $A_C/A_B \approx 2$.

Wall effects and choked flow for lifting bodies have been studied by Cohen and Gilbert (1957), Cohen *et al.* (1957), Fabula (1964), Ai (1966), and others because of their importance to the water tunnel testing of hydrofoils. Moreover, similar phenomena will clearly occur in other internal flow geometries, for example that of a pump impeller. The choked cavitation numbers that emerge from such calculations can be very useful as indicators of the limiting cavitation operation of turbomachines such as pumps and turbines.

Finally, it is appropriate to add some comments on the wall effects in finite cavity flows for which $\sigma > \sigma_c$. It is counterintuitive that the blockage effect should cause a *reduction* in the drag at the same cavitation number as illustrated in section (Nuf). Another remarkable feature of the wall effect, as Wu *et al.* (1971) demonstrate, is that the more streamlined the body the *larger* the fractional change in the drag caused by the wall effect. Consequently, it is *more* important to estimate and correct for the wall effects on streamline bodies than it is for bluff bodies with the same blockage ratio, A_B/A_T. Wu *et al.* (1971) evaluate these wall effects for the planar flows past cavitating wedges of various vertex angles (then $A_B/A_T = b/B$, section (Nuf)) and suggest the following procedure for estimating the drag in the absence of wall effects. If during the experiment one were to measure the minimum coefficient of pressure, C_{pw}, on the tunnel wall at the point opposite the maximum width of the cavity, then Wu et al. recommend use of the following correction rule to estimate the coefficient of drag in the absence of wall effects, $C_D(\sigma', 0)$, from the measured coefficient, $C_D(\sigma, b/B)$. The *effective* cavitation number for the unconfined flow is found to be σ' where

$$\sigma' = \sigma + 2C_{pw}(2-\sigma)/3(1-C_{pw}) \tag{Nud6}$$

and the unconfined drag coefficient is

$$C_D(\sigma', 0) = \frac{(1+\sigma')}{(1+\sigma)} C_D(\sigma, b/B) + O\left(\frac{b^2}{B^2}\right) \tag{Nud7}$$

As illustrated in section (Nuf), Wu *et al.* (1971) use experimental data to show that this correction rule works well for flows around wedges with various vertex angles.