Unsteady Flows

Most of the analyses in the preceding sections addressed various *steady* free streamline flows. The corresponding unsteady flows pose more formidable modeling problems, and it is therefore not surprising that progress in solving these unsteady flows has been quite limited. Though Wang and Wu (1965) show how a general perturbation theory of cavity flows may be formulated, the implementation of their methodology to all but the simplest flows may be prohibitively complicated. Moreover, there remains much uncertainity regarding the appropriate closure model to use in unsteady flow. Consequently, the case of zero cavitation number raises less uncertainty since it involves an infinitely long cavity and no closure. We will therefore concentrate on the linear solution of the problem of small amplitude perturbations to a mean flow with zero cavitation number. This problem was first solved by Woods (1957) in the context of an oscillating aerofoil with separated flow but can be more confidently applied to the cavity flow problem. Martin (1962) and Parkin (1962) further refined Woods' theory and provided tabulated data for the unsteady force coefficients, which we will utilize in this summary.

The unsteady flow problem is best posed using the "acceleration potential" (see, for example, Biot 1942), denoted here by ϕ' and defined simply as $(p_\infty - p)/\rho$, so that linearized versions of Euler's equations of motion may be written as

$$\frac{\partial \phi'}{\partial x} = \frac{\partial u}{\partial t} + U_\infty \frac{\partial u}{\partial x} \tag{Nul1}$$

$$\frac{\partial \phi'}{\partial y} = \frac{\partial v}{\partial t} + U_\infty \frac{\partial v}{\partial x} \tag{Nul2}$$

It follows from the equation of continuity that ϕ' satisfies Laplace's equation,

$$\nabla^2 \phi' = 0 \tag{Nul3}$$

Now consider the boundary conditions on the cavity and on the wetted surface of a flat plate foil. Since the cavity pressure at zero cavitation number is equal to p_∞, it follows that the boundary condition on a free surface is $\phi' = 0$. The linearized condition on a wetted surface is clearly

$$v = -\alpha U_\infty - \frac{\partial h}{\partial t} \tag{Nul4}$$

where $y = -h(x,t)$ describes the geometry of the wetted surface, α is the angle of incidence, and the chord of the foil is taken to be unity. We consider a flat plate at a mean angle of incidence of $\bar{\alpha}$ that is undergoing small-amplitude oscillations in both heave and pitch at a frequency, ω. The amplitude and phase of the pitching oscillations are incorporated in the complex quantity, $\tilde{\alpha}$, so that the instantaneous angle of incidence is given by

$$\alpha = \bar{\alpha} + Re\left\{\tilde{\alpha} e^{j\omega t}\right\} \tag{Nul5}$$

and the amplitude and phase of the heave oscillations of the leading edge are incorporated in the complex quantity $\tilde{h}$ (positive in the negative y direction) so that

$$h(x,t) = Re\left\{\tilde{h} e^{j\omega t}\right\} + xRe\left\{\tilde{\alpha} e^{j\omega t}\right\} \tag{Nul6}$$

where the origin of x is taken to be the leading edge. Combining Equations (**??**), (**??**), and (**??**), the boundary condition on the wetted surface becomes

$$\frac{\partial \phi'}{\partial y} = Re\left\{(\omega^2 \tilde{h} + \omega^2 \tilde{\alpha} x - 2j\omega U_\infty \tilde{\alpha}) e^{j\omega t}\right\} \tag{Nul7}$$

Consequently, the problem reduces to solving for the analytic function $\phi'(z)$ subject to the conditions that ϕ' is zero on a free streamline and that, on a wetted surface, $\partial\phi'/\partial y$ is a known, linear function of x given by equation (Nul7).

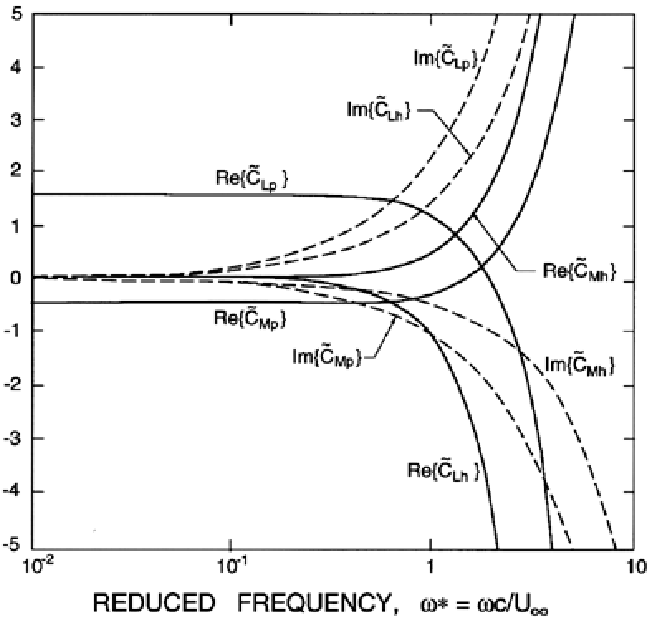

Figure 1: Real and imaginary parts of the four unsteady lift and moment coefficients for a flat plate hydrofoil at zero cavitation number.

In the linearized form this mathematical problem is quite similar to that of the steady flow for a cavitating foil at an angle of attack and can be solved by similar methods (Woods 1957, Martin 1962). The resulting instantaneous lift and moment coefficients can be decomposed into components due to the pitch and the heave:

$$C_L = \bar{C}_L + Re\left\{(\tilde{h}\tilde{C}_{Lh} + \tilde{\alpha}\tilde{C}_{Lp})e^{j\omega t}\right\} \tag{Nul8}$$

$$C_M = \bar{C}_M + Re\left\{(\tilde{h}\tilde{C}_{Mh} + \tilde{\alpha}\tilde{C}_{Mp})e^{j\omega t}\right\} \tag{Nul9}$$

where the moment about the leading edge is considered positive in the clockwise direction (tending to increase α). The four complex coefficients, $\tilde{C}_{Lh}$, $\tilde{C}_{Lp}$, $\tilde{C}_{Mh}$, and $\tilde{C}_{Mp}$, represent the important dynamic characteristics of the foil and are functions of the reduced frequency defined as $\omega^* = \omega c/U_\infty$ where c is the chord. The tabulations by Parkin (1962) allow evaluation of these coefficients, and they are presented in Figure 1 as functions of the reduced frequency. The values tabulated by Woods (1957) yield very similar results. Note that when the reduced frequency is much less than unity, the coefficients tend to their quasistatic values; in this limit all but $Re\{\tilde{C}_{Lp}\}$ and $Re\{\tilde{C}_{Mp}\}$ tend to zero, and these two nonzero coefficients tend to the quasistatic values of $dC_L/d\alpha$ and $dC_M/d\alpha$, namely $\pi/2$ and $5\pi/32$, respectively.

Acosta and DeLong (1971) measured the oscillating forces on a cavitating hydrofoil subjected to heave oscillations at various reduced frequencies. Their results both for cavitating and noncavitating flow are presented in Figure 2 for several mean angles of incidence, $\bar{\alpha}$. The analytical results from figure 1 are included in this figure and compare fairly well with the experiments. Indeed, the agreement is better than is manifest between theory and experiment in the noncavitating case, perhaps because the oscillations of the pressure in the separated region or wake of the noncavitating flow are not adequately modeled.

Other advances in the treatment of unsteady linearized cavity flows were introduced by Wu (1957) and Timman (1958), and the original work of Woods was extended to finite cavitation numbers (finite cavities)

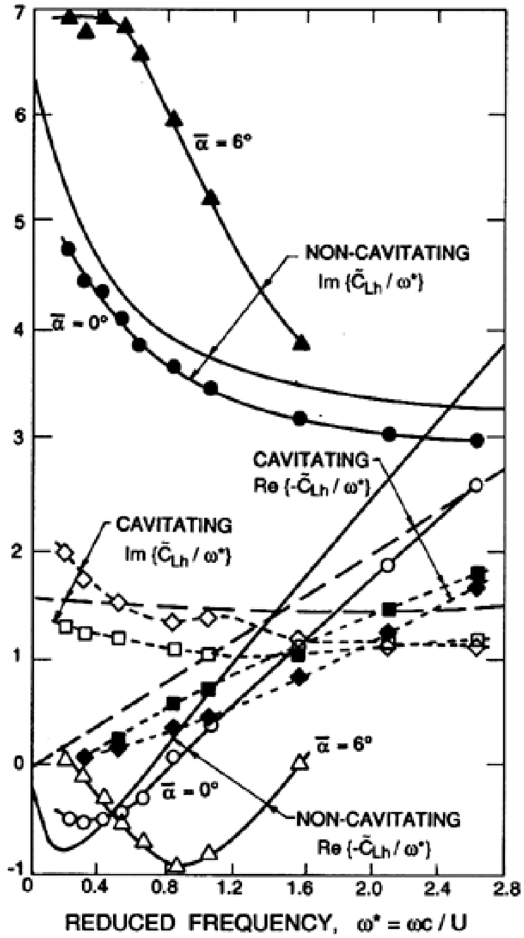

Figure 2: Fluctuating lift coefficients, $\tilde{C}_{Lh}$, for foils undergoing heave oscillations at various reduced frequencies, ω^*. Real and imaginary parts of $\tilde{C}_{Lh}/\omega*$ are presented for noncavitating flow at mean incidence angles of 0° and 6° (solid symbols) and for cavitating flow for a mean incidence of 8°, for very long choked cavities (□) and for cavities 3 chords in length (◇). Adapted from Acosta and DeLong (1971).

by Kelly (1967), who found that the qualitative nature of the results was not dependent on σ. Later, Widnall (1966) showed how the linearized acceleration potential methods could be implemented in three dimensions. Another valuable extension would be to a cascade of foils, but the author is unaware of any similar unsteady data for cavitating cascades. Indeed, apart from the work of Sisto (1967), very little analytical work has been done on the problem of the unsteady response of separated flow in a cascade, a problem that is of considerable importance in the context of turbomachinery. Though progress has been made in understanding the "dynamic stall" of a single foil (see, for example, Ham 1968), there seems to be a clear need for further research on the unsteady behavior of separated and cavitating flows in cascades. The unsteady lift and moment coefficients are not only valuable in determining the unsteady characteristics of propulsion and lift systems but have also been used to predict the flutter and divergence characteristics of cavitating foils (for example, Brennen *et al.* 1980).