# Acoustic Damping

Another important consequence of relative motion is the effect it has on the propagation of plane acoustic waves in a dusty gas. Here we will examine both the propagation velocity and damping of such waves. To do so we postulate a uniform dusty gas and denote the mean state of this mixture by an overbar so that $\bar{p}$, $\bar{T}$, $\bar{\rho}_C$, $\bar{\xi}$ are respectively the pressure, temperature, gas density and mass loading of the uniform dusty gas. Moreover we chose a frame of reference relative to the mean dusty gas so that $\bar{u}_C = \bar{u}_D = 0$. Then we investigate small, linearized perturbations to this mean state denoted by $\tilde{p}$, $\tilde{T}_C$, $\tilde{T}_D$, $\tilde{\rho}_C$, $\tilde{\alpha}_D$, $\tilde{u}_C$, and $\tilde{u}_D$. Substituting into the basic continuity, momentum and energy equations (Nnb1), (Nnb2) and (Nnb3), utilizing the expressions and assumptions of section (Nnd) and retaining only terms linear in the perturbations, the equations governing the propagation of plane acoustic waves become

$$\frac{\partial \tilde{u}_C}{\partial x} + \frac{1}{\bar{p}}\frac{\partial \tilde{p}}{\partial t} - \frac{1}{\bar{T}}\frac{\partial \tilde{T}_C}{\partial t} = 0 \tag{Nnf1}$$

$$\rho_D \frac{\partial \tilde{\alpha}_D}{\partial t} + \frac{\partial \tilde{u}_D}{\partial x} = 0 \tag{Nnf2}$$

$$\frac{\partial \tilde{u}_C}{\partial t} + \frac{\xi \tilde{u}_C}{t_u} - \frac{\xi \tilde{u}_D}{t_u} + \frac{1}{\gamma}\frac{\partial \tilde{p}}{\partial x} = 0 \tag{Nnf3}$$

$$\frac{\partial \tilde{u}_D}{\partial t} + \frac{\tilde{u}_D}{t_u} - \frac{\tilde{u}_C}{t_u} = 0 \tag{Nnf4}$$

$$\frac{\partial \tilde{T}_C}{\partial t} + \frac{\xi \tilde{T}_C}{t_T} - \frac{\xi \tilde{T}_D}{t_T} + \frac{(\gamma-1)\bar{p}}{\gamma \bar{T}}\frac{\partial \tilde{p}}{\partial t} = 0 \tag{Nnf5}$$

$$\frac{\partial \tilde{T}_D}{\partial t} + \frac{c_{pC}\tilde{T}_D}{c_{sD}t_T} - \frac{c_{pC}\tilde{T}_C}{c_{sD}t_T} = 0 \tag{Nnf6}$$

where $\gamma = c_{pC}/c_{vC}$. Note that the particle volume fraction perturbation only occurs in one of these, equation (Nnf2); consequently this equation may be set aside and used after the solution has been obtained in order to calculate $\tilde{\alpha}_D$ and therefore the perturbations in the particle loading $\tilde{\xi}$. The basic form of a plane acoustic wave is

$$Q(x,t) = \bar{Q} + \tilde{Q}(x,t) = \bar{Q} + Re\left\{Q(\omega)e^{i\kappa x + i\omega t}\right\} \tag{Nnf7}$$

where $Q(x,t)$ is a generic flow variable, $\omega$ is the acoustic frequency and $\kappa$ is a complex function of $\omega$; clearly the phase velocity of the wave, $c_\kappa$, is given by $c_\kappa = Re\{-\omega/\kappa\}$ and the non-dimensional attenuation is given by $Im\{-\kappa\}$. Then substitution of the expressions (Nnf7) into the five equations (Nnf1), (Nnf3), (Nnf4), (Nnf5) and (Nnf6) yields the following dispersion relation for $\kappa$:

$$\left(\frac{\omega}{\kappa c_C}\right)^2 = \frac{(1+i\omega t_u)(\frac{c_{pC}}{c_{sD}} + \xi + i\omega t_T)}{(1+\xi+i\omega t_u)(\frac{c_{pC}}{c_{sD}}\gamma\xi + i\omega t_T)} \tag{Nnf8}$$

where $c_C = (\gamma \mathcal{R}_C \bar{T})^{\frac{1}{2}}$ is the speed of sound in the gas alone. Consequently, the phase velocity is readily obtained by taking the real part of the square root of the right hand side of equation (Nnf8). It is a function of frequency, $\omega$, as well as the relaxation times, $t_u$ and $t_T$, the loading, $\xi$, and the specific heat ratios, $\gamma$ and $c_{pC}/c_{sD}$. Typical results are shown in figures 1 and 2.

The mechanics of the variation in the phase velocity (acoustic speed) are evident by inspection of equation (Nnf8) and figures 1 and 2. At very low frequencies such that $\omega t_u \ll 1$ and $\omega t_T \ll 1$, the velocity and

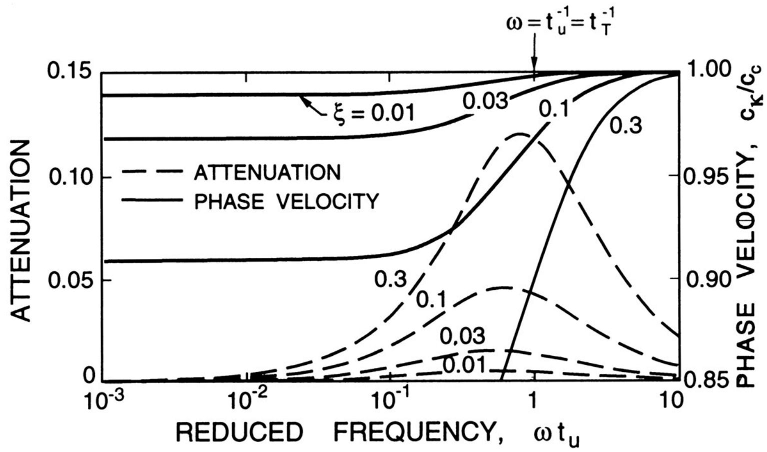


Figure 1: Non-dimensional attenuation, $Im\{-\kappa c_C/\omega\}$ (dotted lines), and phase velocity, $c_\kappa/c_C$ (solid lines), as functions of reduced frequency, $\omega t_u$, for a dusty gas with various loadings, $\xi$, as shown and $\gamma = 1.4$, $t_T/t_u = 1$ and $c_{pC}/c_{sD} = 0.3$.

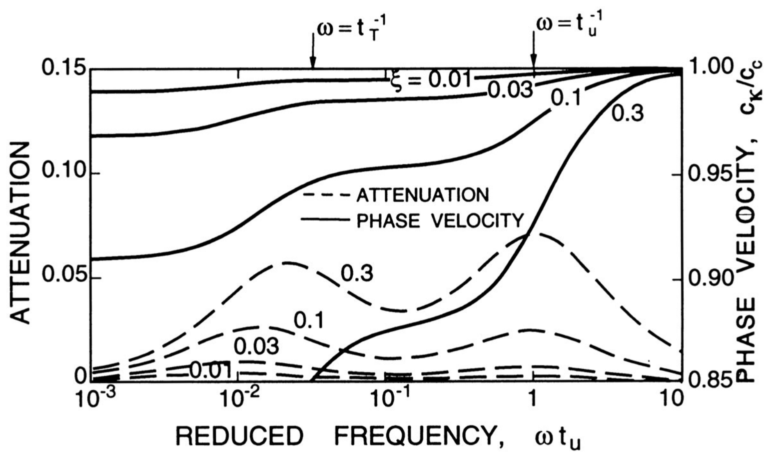


Figure 2: Non-dimensional attenuation, $Im\{-\kappa c_C/\omega\}$ (dotted lines), and phase velocity, $c_\kappa/c_C$ (solid lines), as functions of reduced frequency, $\omega t_u$, for a dusty gas with various loadings, $\xi$, as shown and $\gamma = 1.4$, $t_T/t_u = 30$ and $c_{pC}/c_{sD} = 0.3$.

temperature relaxations are essentially instantaneous. Then the phase velocity is simply obtained from the effective properties and is given by equation (Nnc8). These are the phase velocity asymptotes on the left-hand side of figures 1 and 2. On the other hand, at very high frequencies such that $\omega t_u \gg 1$ and $\omega t_T \gg 1$, there is negligible time for the particles to adjust and they simply do not participate in the propagation of the wave; consequently, the phase velocity is simply the acoustic velocity in the gas alone, $c_C$. Thus all phase velocity lines asymptote to unity on the right in the figures. Other ranges of frequency may also exist (for example $\omega t_u \gg 1$ and $\omega t_T \ll 1$ or the reverse) in which other asymptotic expressions for the acoustic speed can be readily extracted from equation (Nnf8). One such intermediate asymptote can be detected in figure 2. It is also clear that the acoustic speed decreases with increased loading, $\xi$, though only weakly in some frequency ranges. For small $\xi$ the expression (Nnf8) may be expanded to obtain the linear change in the acoustic speed with loading, $\xi$, as follows:

$$\frac{c_\kappa}{c_C} = 1 - \frac{\xi}{2}\left[\frac{(\gamma-1)\frac{c_{pC}}{c_{sD}}}{\left\{(c_{pC}/c_{sD})^2 + (\omega t_T)^2\right\}} + \frac{1}{\{1+(\omega t_T)^2\}}\right] + .... \tag{Nnf9}$$

This expression shows why, in figures 1 and 2, the effect of the loading, $\xi$, on the phase velocity is small at higher frequencies.

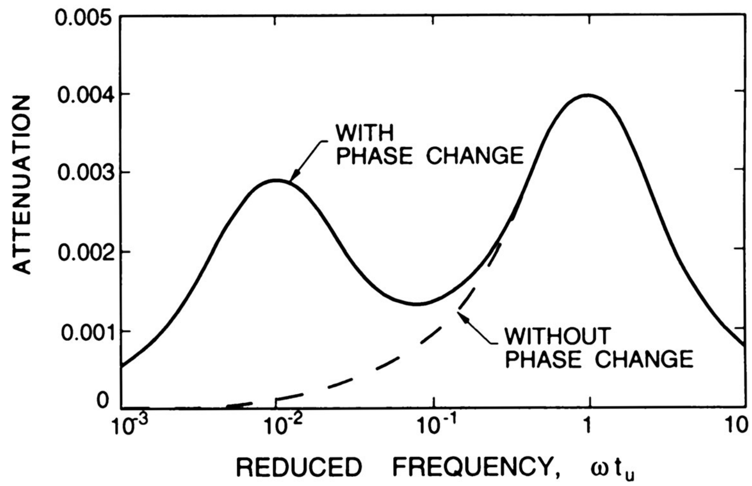


Figure 3: Non-dimensional attenuation, $Im\{-\kappa c_C/\omega\}$, as a function of reduced frequency for a droplet-laden gas flow with $\xi = 0.01$, $\gamma = 1.4$, $t_T/t_u = 1$ and $c_{pC}/c_{sD} = 1$. The dashed line is the result without phase change; the solid line is an example of the alteration caused by phase change. Adapted from Marble and Wooten (1970).

Now we examine the attenuation manifest in the dispersion relation (Nnf8). The same expansion for small $\xi$ that led to equation (Nnf9) also leads to the following expression for the attenuation:

$$Im\{-\kappa\} = \frac{\xi\omega}{2c_C}\left[\frac{(\gamma-1)\omega t_T}{\left\{(c_{pC}/c_{sD})^2 + (\omega t_T)^2\right\}} + \frac{\omega t_u}{\{1+(\omega t_T)^2\}}\right] + .... \tag{Nnf10}$$

In figures 1 and 2, a dimensionless attenuation, $Im\{-\kappa c_C/\omega\}$, is plotted against the reduced frequency. This particular non-dimensionalization is somewhat misleading since, plotted without the $\omega$ in the denominator, the attenuation increases monotonically with frequency. However, this presentation is commonly used to demonstrate the enhanced attenuations that occur in the neighborhoods of $\omega = t_u^{-1}$ and $\omega = t_T^{-1}$ and which are manifest in figures 1 and 2.

When the gas contains liquid droplets rather than solid particles, the same basic approach is appropriate except for the change that might be caused by the evaporation and condensation of the liquid during the passage of the wave. Marble and Wooten (1970) present a variation of the above analysis that includes the effect of phase change and show that an additional maximum in the attenuation can result as illustrated in figure 3. This additional peak results from another relaxation process embodied in the phase change process. As Marble (1970) points out it is only really separate from the other relaxation times when the loading is small. At higher loadings the effect merges with the velocity and temperature relaxation processes.