Example: Horizontal Pipe Flow

As a quantitative example, we shall pursue the case of the flow of a two-component mixture in a long horizontal pipe. The separation velocity, W_p, due to gravity, g, would then be given qualitatively by equation (Nei3) or (Nei12), namely

$$W_p = \frac{2R^2 g}{9\nu_C}\left(\frac{\Delta\rho}{\rho_C}\right) \quad \text{if} \quad 2W_pR/\nu_C \ll 1$$

or

$$W_p = \left\{\frac{2}{3}\frac{Rg}{C_D}\frac{\Delta\rho}{\rho_C}\right\}^{\frac{1}{2}} \quad \text{if} \quad 2W_pR/\nu_C \gg 1 \tag{Njg1}$$

where R is the particle, droplet, or bubble radius, ν_C, ρ_C are the kinematic viscosity and density of the continuous fluid, and $\Delta\rho$ is the density difference between the components. Furthermore, the typical turbulent velocity will be some function of the friction velocity, $(\tau_w/\rho_C)^{\frac{1}{2}}$, and the volume fraction, α, of the disperse phase. The effect of α is less readily quantified so, for the present, we concentrate on dilute systems ($\alpha \ll 1$) in which

$$W_t \approx \left(\frac{\tau_w}{\rho_C}\right)^{\frac{1}{2}} = \left\{\frac{d}{4\rho_C}\left(-\frac{dp}{ds}\right)\right\}^{\frac{1}{2}} \tag{Njg2}$$

where d is the pipe diameter and dp/ds is the pressure gradient. Then the transition condition, $W_p/W_t = K$ (where K is some number of order unity) can be rewritten as

$$\left(-\frac{dp}{ds}\right) \approx \frac{4\rho_C}{K^2 d}W_p^2$$

$$\approx \frac{16}{81K^2}\frac{\rho_C R^4 g^2}{\nu_C^2 d}\left(\frac{\Delta\rho}{\rho_C}\right)^2 \quad \text{for} \quad 2W_pR/\nu_C \ll 1 \tag{Njg3}$$

$$\approx \frac{32}{3K^2}\frac{\rho_C Rg}{C_D d}\left(\frac{\Delta\rho}{\rho_C}\right) \quad \text{for} \quad 2W_pR/\nu_C \gg 1 \tag{Njg4}$$

In summary, the expression on the right hand side of equation (Njg3) (or (Njg4)) yields the pressure drop at which W_p/W_t exceeds the critical value of K and the particles will be maintained in suspension by the turbulence. At lower values of the pressure drop the particles will settle out and the flow will become separated and stratified.

This criterion on the pressure gradient may be converted to a criterion on the flow rate by using some version of the turbulent pipe flow relation between the pressure gradient and the volume flow rate, j. For example, one could conceive of using, as a first approximation, a typical value of the turbulent friction factor, $f = \tau_w/\frac{1}{2}\rho_C j^2$ (where j is the total volumetric flux). In the case of $2W_pR/\nu_C \gg 1$, this leads to a critical volume flow rate, $j = j_c$, given by

$$j_c = \left\{\frac{8}{3K^2 f}\frac{gD}{C_D}\frac{\Delta\rho}{\rho_C}\right\}^{\frac{1}{2}} \tag{Njg5}$$

With $8/3K^2 f$ replaced by an empirical constant, this is the general form of the critical flow rate suggested by Newitt *et al.* (1955) for horizontal slurry pipeline flow; for $j > j_c$ the flow regime changes from saltation flow to heterogeneous flow (see section (Njc)). Alternatively, one could write this nondimensionally using a Froude number defined as $Fr = j_c/(gd)^{\frac{1}{2}}$. Then the criterion yields a critical Froude number given by

$$Fr^2 = \frac{8}{3K^2 f C_D} \frac{\Delta\rho}{\rho_C} \tag{Njg6}$$

If the common expression for the turbulent friction factor, namely $f = 0.31/(jd/\nu_C)^{\frac{1}{4}}$ is used in equation (Njg5), that expression becomes

$$j_c = \left\{ \frac{17.2}{K^2 C_D} \frac{gRd^{\frac{1}{4}}}{\nu_C^{\frac{1}{4}}} \frac{\Delta\rho}{\rho_C} \right\}^{\frac{4}{7}} \tag{Njg7}$$

A numerical example will help relate this criterion (Njg7) to the boundary of the disperse phase regime in the flow regime maps. For the case of figure 3 of section (Njb) and using for simplicity, $K = 1$ and $C_D = 1$, then with a drop or bubble size, $R = 3mm$, equation (Njg7) gives a value of j_c of $3m/s$ when the continuous phase is liquid (bubbly flow) and a value of $40m/s$ when the continuous phase is air (mist flow). These values are in good agreement with the total volumetric flux at the boundary of the disperse flow regime in figure 3, section (Njb) which, at low j_G, is about $3m/s$ and at higher j_G (volumetric qualities above 0.5) is about $30 - 40m/s$.

Another approach to the issue of the critical velocity in slurry pipeline flow is to consider the velocity required to fluidize a packed bed in the bottom of the pipe (see, for example, Durand and Condolios (1952) or Zandi and Govatos (1967)). This is described further in section (Nkd).