Heterogeneous Flow Friction

The most substantial remaining issue is to understand the much larger friction factors that occur when particle segregation predominates. For example, commenting on the data of figure 2, section (Nkb), Lazarus and Neilsen show that values larger than the base curves begin when component separation begins to occur and the flow regime changes from the heterogeneous regime to the saltation regime (section (Njc)). Another slurry flow example is shown in figure 1. According to Hayden and Stelson (1971) the minima in the fitted curves correspond to the boundary between the heterogeneous and saltation flow regimes. Note that these all occur at essentially the same critical volumetric flux, j_c; this agrees with the criterion of Newitt *et al.* (1955) that was discussed in section (Njf) and is equivalent to a critical volumetric flux, j_c, that is simply proportional to the terminal velocity of individual particles and independent of the loading or mass fraction.

The transition of the flow regime from heterogeneous to saltation results in much of the particle mass being supported directly by particle contacts with the interior surface of the pipe. The frictional forces that this contact produces implies, in turn, a substantial pressure gradient in order to move the bed. The pressure gradient in the moving bed configuration can be readily estimated as follows. The submerged weight of solids in the packed bed per unit length of the cylindrical pipe of diameter, d, is

$$\pi d^2 \alpha g(\rho_S - \rho_L) \qquad \text{(Nkd1)}$$

where α is the overall effective volume fraction of solids. Therefore, if the effective Coulomb friction coefficient is denoted by η, the longitudinal force required to overcome this friction per unit length of pipe is simply η times the above expression. The pressure gradient needed to provide this force is therefore

$$-\left(\frac{dp}{ds}\right)_{friction} = \eta \alpha g(\rho_S - \rho_L) \qquad \text{(Nkd2)}$$

With η considered as an adjustable constant, this is the expression for the additional frictional pressure gradient proposed by Newitt *et al.* (1955). The final step is to calculate the volumetric flow rate that

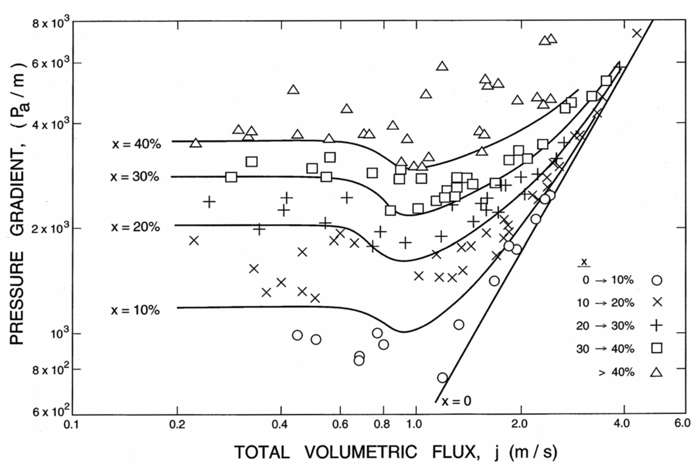

Figure 1: Pressure gradients in a 2.54cm diameter horizontal pipeline plotted against the total volumetric flux, j, for a slurry of sand with particle diameter 0.057cm. Curves for four specific mass fractions, x (in percent) are fitted to the data. Adapted from Hayden and Stelson (1971).

occurs with this pressure gradient, part of which proceeds through the packed bed and part of which flows above the bed. The literature contains a number of semi-empirical treatments of this problem. One of the first correlations was that of Durand and Condolios (1952) that took the form

$$j_c = f(\alpha, D)\left\{2gd\frac{\Delta\rho}{\rho_L}\right\}^{\frac{1}{2}} \tag{Nkd3}$$

where $f(\alpha, D)$ is some function of the solids fraction, α, and the particle diameter, D. There are both similarities and differences between this expression and that of Newitt *et al.* (1955). A commonly used criterion that has the same form as equation (Nkd3) but is more specific is that of Zandi and Govatos (1967):

$$j_c = \left\{\frac{K\alpha dg}{C_D^{\frac{1}{2}}}\frac{\Delta\rho}{\rho_L}\right\}^{\frac{1}{2}} \tag{Nkd4}$$

where K is an empirical constant of the order of $10-40$. Many other efforts have been made to correlate the friction factor for the heterogeneous and saltation regimes; reviews of these mostly empirical approaches can be found in Zandi (1971) and Lazarus and Neilsen (1978). Fundamental understanding is less readily achieved; perhaps future understanding of the granular flows described in sections (Np) will provide clearer insights.