Vertical Pipe Flow

Consider first the vertical pipe flow of two generic components, A and B. For ease of visualization, we consider

- that vertically upward is the positive direction so that all fluxes and velocities in the upward direction are positive
- that A is the less dense component and, as a memory aid, we will call A the gas and denote it by A = G. Correspondingly, the denser component B will be termed the liquid and denoted by B = L.
- that, for convenience, $\alpha = \alpha_G = 1 \alpha_L$.

However, any other choice of components or relative densities are readily accommodated in this example by simple changes in these conventions. We shall examine the range of phenomena exhibited in such a flow by the somewhat artificial device of fixing the gas flux, j_G , and varying the liquid flux, j_L . Note that in this context equation (Nqb1) becomes

$$j_{GL} = (1 - \alpha)j_G - \alpha j_L \tag{Nqc1}$$



Figure 1: Drift flux charts for the vertical flows of gas-liquid mixtures. Left: for downward gas flux. Right: for upward gas flux.

Consider, first, the case of downward or negative gas flux as shown on the left in figure 1. When the liquid flux is also downward the operating point, OP, is usually well defined as illustrated by CASE A in figure 1. However, as one might anticipate, it is impossible to have an upward flux of liquid with a downward flux of gas and this is illustrated by the fact that CASE B has no intersection point and no solution.

The case of upward or positive gas flux, shown on the right in figure 1, is more interesting. For downward liquid flux (CASE C) there is usually just one, unambiguous, operating point, OP. However, for small upward liquid fluxes (CASE D) we see that there are two possible solutions or operating points, OP1 and OP2. Without more information, we have no way of knowing which of these will be manifest in a particular application. In mathematical terms, these two operating points are known as *conjugate states*. Later we shall see that structures known as *kinematic shocks* or expansion waves may exist and allow transition of



Figure 2: Flooding envelope in a flow pattern diagram.

the flow from one conjugate state to the other. In many ways, the situation is analogous to gasdynamic flows in pipes where the conjugate states are a subsonic flow and a supersonic flow or to open channel flows where the conjugate states are a subcritical flow and a supercritical flow. The structure and propagation of kinematic waves and shocks are will be discussed later in sections (Ns).

One further phenomenon manifests itself if we continue to increase the downward flux of liquid while maintaining the same upward flux of gas. As shown on the right in figure 1, we reach a limiting condition (CASE F) at which the dashed line becomes tangent to the drift flux curve at the operating point, OPF. We have reached the maximum downward liquid flux that will allow that fixed upward gas flux to move through the liquid. This is known as a *flooded* condition and the point OPF is known as the flooding point. As the reader might anticipate, flooding is quite analogous to choking and might have been better named choking to be consistent with the analogous phenomena that occur in gasdynamics and in open-channel flow.

It is clear that there exists a family of flooding conditions that we shall denote by j_{Lf} and j_{Gf} . Each member of this family corresponds to a different tangent to the drift flux curve and each has a different volume fraction, α . Indeed, simple geometric considerations allow one to construct the family of flooding conditions in terms of the parameter, α , assuming that the drift flux function, $j_{GL}(\alpha)$, is known:

$$j_{Gf} = j_{GL} - \alpha \frac{dj_{GL}}{d\alpha} \quad ; \quad j_{Lf} = -j_{GL} - (1 - \alpha) \frac{dj_{GL}}{d\alpha} \tag{Nqc2}$$

Often, these conditions are displayed in a flow regime diagram (see sections (Nj)) in which the gas flux is plotted against the liquid flux. An example is shown in figure 2. In such a graph it follows from the basic relation (Nqc2) (and the assumption that j_{GL} is a function only of α) that a contour of constant void fraction, α , will be a straight line similar to the dashed lines in figure 2. The slope of each of these dashed lines is $\alpha/(1-\alpha)$, the intercept with the j_G axis is $j_{GL}/(1-\alpha)$ and the intercept with the j_L axis is $-j_{GL}/\alpha$. It is then easy to see that these dashed lines form an envelope, AB, that defines the flooding conditions in this flow regime diagram. No flow is possible in the fourth quadrant and above and to the left of the flooding envelope. Note that the end points, A and B, may yield useful information. In the case of the drift flux given by equation (Nqa2), the points A and B are given respectively by

$$(j_G)_A = u_{GL0}(1-b)^{1-b}/b^b$$
; $(j_L)_B = -u_{GL0}$ (Nqc3)

Finally we note that since, in mathematical terms, the flooding curve in figure 2 is simply a mapping of the drift flux curve in figure 1, it is clear that one can construct one from the other and vice-versa. Indeed, one

of the most convenient experimental methods to determine the drift flux curve is to perform experiments at fixed void fractions and construct the dashed curves in figure 2. These then determine the flooding envelope from which the drift flux curve can be obtained.