Bubble Growth by Mass Diffusion

In most of the circumstances considered in this chapter, it is assumed that the events occur too rapidly for significant mass transfer of contaminant gas to occur between the bubble and the liquid. Thus we assumed in section (Ngc) and elsewhere that the mass of contaminant gas in the bubble remained constant. It is convenient to reconsider this issue at this point, for the methods of analysis of mass diffusion will clearly be similar to those of thermal diffusion as described in section (Ngc) (see Scriven 1959). Moreover, there are some issues that require analysis of the rate of increase or decrease of the mass of gas in the bubble. One of the most basic issues is the fact that any and all of the gas-filled microbubbles that are present in a subsaturated liquid (and particularly in water) should dissolve away if the ambient pressure is sufficiently high. Henry's law states that the partial pressure of gas, p_{Ge}, in a bubble that is in equilibrium with a saturated concentration, c_∞, of gas dissolved in the liquid will be given by

$$p_{Ge} = c_\infty He \tag{Ngi1}$$

where He is Henry's law constant for that gas and liquid combination (He decreases substantially with temperature). Consequently, if the ambient pressure, p_∞, is greater than $(c_\infty He + p_V - 2S/R)$, the bubble should dissolve away completely. Experience is contrary to this theory, and microbubbles persist even when the liquid is subjected to several atmospheres of pressure for an extended period; in most instances, this stabilization of nuclei is caused by surface contamination.

The process of mass transfer can be analysed by noting that the concentration, $c(r,t)$, of gas in the liquid will be governed by a diffusion equation identical in form to equation (Ngc5),

$$\frac{\partial c}{\partial t} + \frac{dR}{dt}\left(\frac{R}{r}\right)^2 \frac{\partial c}{\partial r} = \frac{D}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right) \tag{Ngi2}$$

where D is the mass diffusivity, typically $2 \times 10^{-5}\ cm^2/sec$ for air in water at normal temperatures. As Plesset and Prosperetti (1977) demonstrate, the typical bubble growth rates due to mass diffusion are so slow that the convection term (the second term on the left-hand side of equation (Ngi2)) is negligible.

The simplest problem is that of a bubble of radius, R, in a liquid at a fixed ambient pressure, p_∞, and gas concentration, c_∞. In the absence of inertial effects the partial pressure of gas in the bubble will be p_{Ge} where

$$p_{Ge} = p_\infty - p_V + 2S/R \tag{Ngi3}$$

and therefore the concentration of gas at the liquid interface is $c_s = p_{Ge}/He$. Epstein and Plesset (1950) found an approximate solution to the problem of a bubble in a liquid initially at uniform gas concentration, c_∞, at time, $t = 0$, that takes the form

$$R\frac{dR}{dt} = \frac{D}{\rho_G}\frac{\{c_\infty - c_s(1+2S/Rp_\infty)\}}{(1+4S/3Rp_\infty)}\left\{1 + R(\pi Dt)^{-\frac{1}{2}}\right\} \tag{Ngi4}$$

where ρ_G is the density of gas in the bubble and c_s is the saturated concentration at the interface at the partial pressure given by equation (Ngi3) (the vapor pressure is neglected in their analysis). The last term in equation (Ngi4), $R(\pi Dt)^{-\frac{1}{2}}$, arises from a growing diffusion boundary layer in the liquid at the bubble surface. This layer grows like $(Dt)^{\frac{1}{2}}$. When t is large, the last term in equation (Ngi4) becomes small and the characteristic growth is given approximately by

$$\{R(t)\}^2 - \{R(0)\}^2 \approx \frac{2D(c_\infty - c_s)t}{\rho_G} \tag{Ngi5}$$

where, for simplicity, we have neglected surface tension.

It is instructive to evaluate the typical duration of growth (or shrinkage). From equation (Ngi5) the time required for complete solution is t_{cs} where

$$t_{cs} \approx \frac{\rho_G \left\{R(0)\right\}^2}{2D(c_s - c_\infty)} \tag{Ngi6}$$

Typical values of $(c_s - c_\infty)/\rho_G$ are 0.01 (Plesset and Prosperetti 1977). Thus, in the absence of surface contaminant effects, a $10\mu m$ bubble should completely dissolve in about $2.5s$.

Finally we note that there is an important mass diffusion effect caused by ambient pressure oscillations in which nonlinearities can lead to bubble growth even in a subsaturated liquid. This is known as *rectified diffusion* and is discussed in section (Ngl).