Auto- and Cross-Correlation

Cross-correlation is the technique used to uncover the relationship between two signals which may be displaced in time. For example, if a particle were to pass by two proximity probes with a time delay between the two then cross-correlation is the signal processing technique used to evaluate that time delay. Knowing the spatial separation of the two probes one could then evaluate the speed of the particle. The

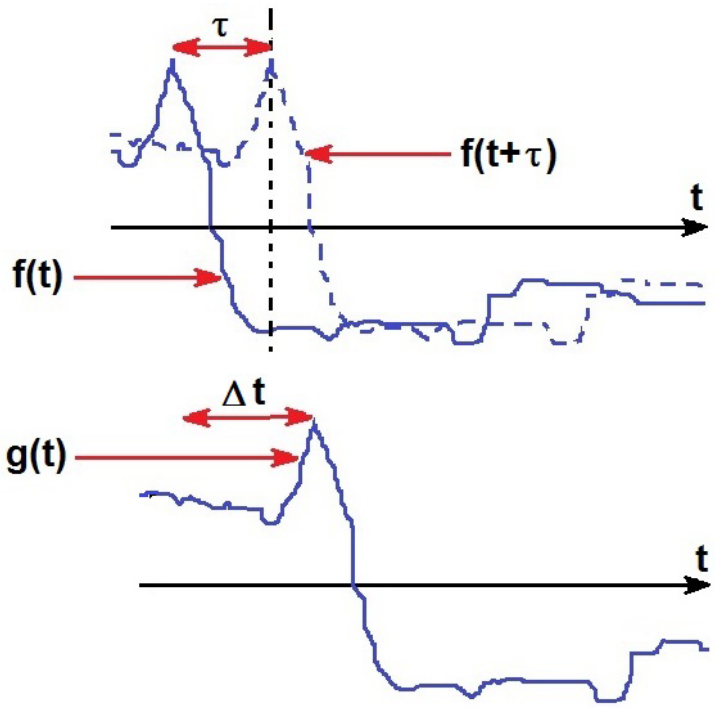

Figure 1: Cross-correlating two signals, $f(t)$ and $g(t)$.

two recorded signals will be denoted by $f(t)$ and $g(t)$ as illustrated in Figure 1. Then, for an extended interval of time, T, we compute the cross-correlation function, $C(\tau)$, defined by

$$C(\tau) \;=\; \int_0^T f(t+\tau)g(t)\;d\tau \tag{Kcb1}$$

for a whole series of values of the variable, τ. This cross-correlation function might look like that plotted in Figure 2. It will be a maximum when $\tau = \Delta t$ where, as depicted in Figure 1, Δt is the time shift at which the two signals are most alike. Thus, by examining the function $C(\tau)$ and finding the maximum (or maximums) the value of Δt can be determined. For example, in the example quoted above, the particle velocity can then be determined from $\ell/\Delta t$ where ℓ is the distance between the two probes. Note that the interval T should be extended until further extension no longer effects the result.

Auto-correlation is a very similar technique used examine the content of one particular signal, $f(t)$. That signal is correlated with the same signal shifted in time, $f(t+\tau)$, by evaluating the integral

$$A(\tau) \;=\; \int_0^T f(t+\tau)f(t)\;d\tau \tag{Kcb2}$$

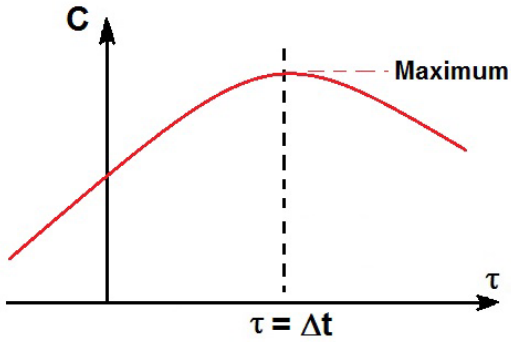

Figure 2: Cross-correlation function, $C(\tau)$.

For example, it is easy to see that a sinusoidal signal with period, T, will exhibit sharp peaks in the auto-correlation function at $\tau = n\pi$. Thus an examination of the auto-correlation function reveals the periodic content in the signal.

There are many other signal processing techniques which are beyond the scope of this book. The reader is referred to Papoulis (1965) for an excellent review of these methods.