Pressure Differences due to Surface Tension

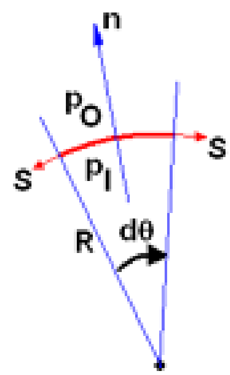

Figure 1: A small section of curved surface (red) with surface tension, S, and pressures, p_O and p_I on the outside and inside respectively.

One common manifestation of surface tension is the difference in pressure it causes across a curved surface. For simplicity we consider first a liquid surface which is curved in only one plane but is flat in a direction perpendicular to that plane. A small section of such a surface is sketched in Figure 1. The pressures in the different fluids on either side of this interface are denoted by p_O (on the outside of curve) and p_I (on the inside of the curve). Now consider all the forces acting on a small section of the surface of length, ds, and unit dimension normal to the sketch. The radius of curvature of the surface is denoted by R as indicated so that the angle subtended at the center of curvature, $d\theta$, is given by $ds = Rd\theta$. Now consider all the forces in the direction n normal to the center of the fluid element. By the result described in **static forces** the net force in the outward direction due to forces near and at a fluid surface that we will describe in this and some linked sections. p_O and p_I will be $2R(p_I - p_O)\sin d\theta$. Opposing this will be the components of the surface tension forces S acting on the two ends of the section of surface which yield an inward force (in the negative n direction) equal to $2S \sin d\theta$. Thus in equilibrium

$$2R(p_I - p_O)\sin d\theta = 2S \sin d\theta \tag{Cm1}$$

or

$$p_I - p_O = S/R \tag{Cm2}$$

Thus the surface tension causes a greater pressure inside the surface and the difference is the surface tension divided by the radius of curvature (so that a flat surface yields no pressure difference). Note that it makes no difference whether the liquid is on the outside or on the inside.

Now consider a spherical surface, specifically the spherical drop or bubble shown in Figure 2. If the drop is cut in half as shown the force imposed by surface tension on the remaining half will be $2\pi RS$. Opposing that will be the pressure difference $(p_I - p_O)$ acting on the projected area πR^2 and therefore, in equilibrium,

$$2\pi RS = \pi R^2(p_I - p_O) \tag{Cm3}$$

so that, in the case of a spherical surface,

$$p_I - p_O = 2S/R \tag{Cm4}$$

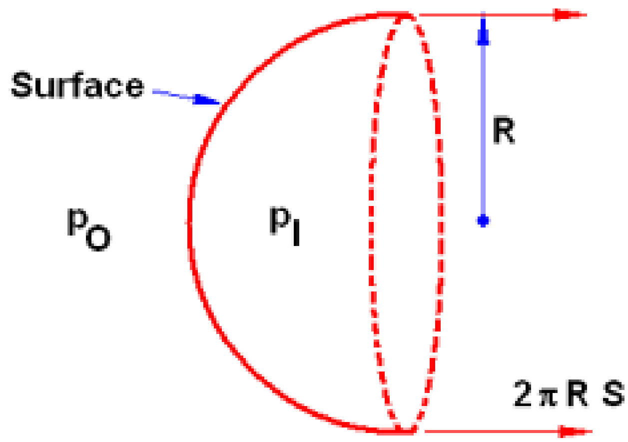

Figure 2: Half of a spherical drop of radius, R, (red) with surface tension, S, and pressures, p_O and p_I on the outside and inside respectively.

An appropriate way to visualize this is that, on the surface, the curvatures in each of the two perpendicular directions contribute equally to the pressure difference caused by the surface tension. Indeed, a general three-dimensional surface will have two principal radii of curvature, R_1 and R_2, and it can be shown that the resulting pressure difference in this general case is given by

$$p_I - p_O = S\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \tag{Cm5}$$

In the special case of the cylindrical surfaces, $R_1 = R$ and $R_2 = \infty$ and in the special case of the sphere, $R_1 = R_2 = R$.