Oceans at rest

In an ocean in which the liquid is essentially at rest the pressure, p, will vary with depth according to the relation derived in the section on **fluid statics** namely

$$p = p_0 - \rho g(y - y_0) \tag{Cc1}$$

where ρ is the liquid density (assumed approximately independent of depth), g is the acceleration due to gravity and p_0 is the reference pressure at some reference location, $y = y_0$. It is convenient to define the surface of the ocean, $y_0 = 0$, as the origin and therefore set p_0 to be the atmospheric pressure at the surface; also recall that y is measured vertically upward so that $(-y)$ is the depth in meters below the surface.

Using the above result we might then ask whether the density is, indeed, approximately independent of depth. At normal temperatures and pressures the above equation yields

$$\frac{p}{p_0} = 1 + 0.098(-y) \tag{Cc2}$$

so that, at a depth of $10m$, the pressure is approximately $2atm$, at a depth of $100m$ it is approximately $11atm$, at a depth of $1000m$ it is roughly $99atm$ and at $10,000m$ it is roughly $981atm$. Using the known bulk modulus of **compressibility** for water of $\kappa = 2206MPa$ we can approximately compute the change in the density, $\Delta\rho$ that would result from these changes in pressure, $(p - p_0) = \Delta p$ using

$$\frac{\Delta\rho}{\Delta p} \approx \frac{\partial\rho}{\partial p} = \frac{\rho}{\kappa} \tag{Cc3}$$

or

$$\frac{\Delta\rho}{\rho} \approx \frac{(p - p_0)}{\kappa} \tag{Cc4}$$

Consequently the percentage reduction in density will be 0.009%, 0.049%, 0.45%, and 4.5% at depths of $10m$, $100m$, $1000m$, and $1000m$ respectively. Thus, for most purposes, oceans and lakes have essentially uniform density except at the deepest depths. In man-made reservoirs the density variations are rarely significant.