Manometers

One of the most widespread uses of the principles of **Fluid Statics** is in manometers, a basic technique for the measurement of pressure. Perhaps the simplest implementation of this technique is in the measurement of the difference in the pressures in two tanks of gas as sketched in Figure 1. Then the pressure

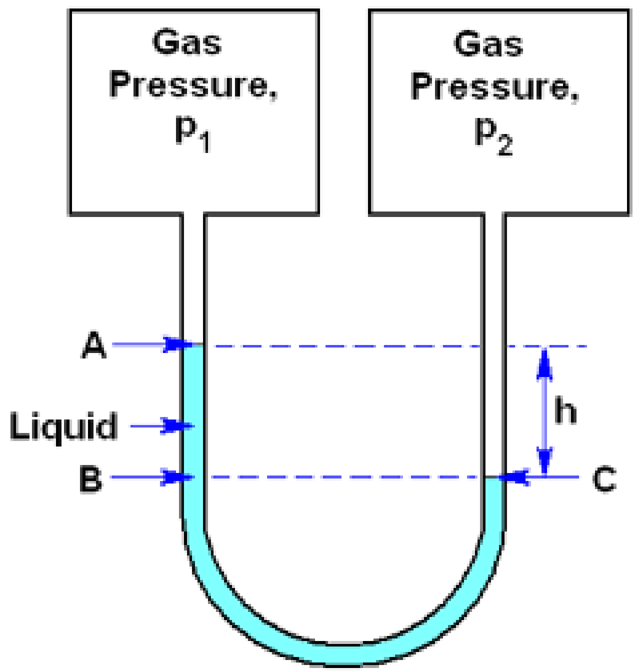

Figure 1: Basic liquid manometer for measuring gas pressure differnces.

difference, $(p_2 - p_1)$, in the two gas tanks generates a difference, h, in the liquid levels on the two sides of the manometer or U-tube where

$$p_2 - p_1 = \rho g h \tag{Cd1}$$

where ρ is the liquid density. Consequently by measuring h accurately and knowing the liquid density we can calculate the required pressure difference. As in all of the other examples which follow the best way to arrive at the characteristic for the manometer is to trace the pressure around the fluid path. Thus the pressure at the point A is p_1 (note that in this example we are neglecting any gravitational variation in the gas pressure since the density of the gas is very small) and it follows that the pressure at the point B must be $(p_1 + \rho g h)$. Since the point C is on the same horizontal plane as the point B the pressure there must be the same and it must also, of course, be equal to p_2. Hence equation (Cd1) follows.

Notice that in addition to neglecting the gravitational variations in the gas pressure, we have also neglected any **pressure differences across the menisci** at the points A and C. This second approximation is a serious problem in the design and use of manometers and there are several ways to mitigate it. One way is to use vertical tubes with a large enough internal diameter that any potential pressure difference is minimized. Another is to use vertical tubes with very accurately controlled internal diameters (and surface contact angles) so that the pressure differences due to **surface tension** at the points A and C are the same. This would then still lead to the result (Cd1).

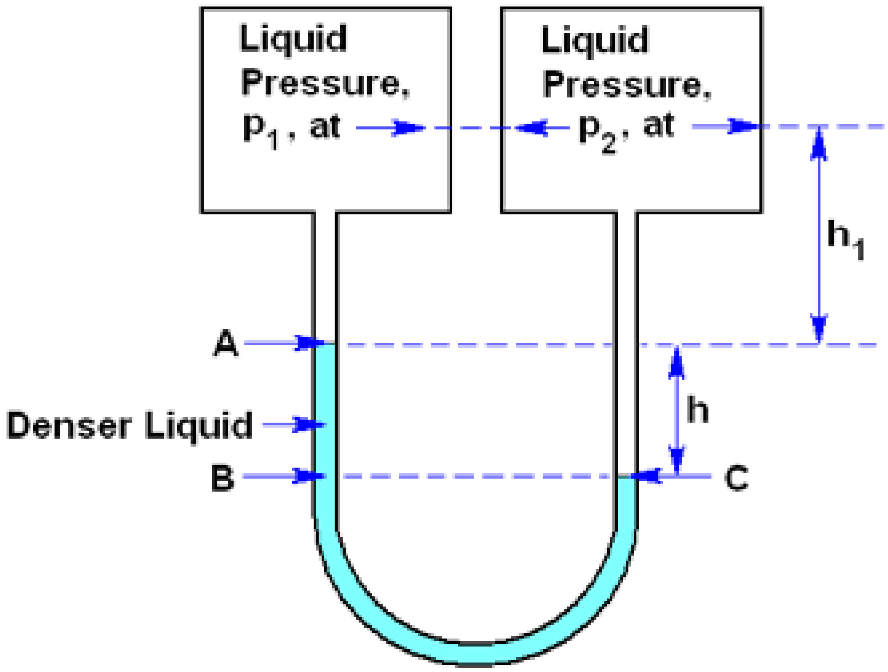

Figure 2: Basic liquid manometer for measuring liquid pressure differnces.

Another important consideration is to measure h as accurately as possible and, for this reason, laboratory manometers, are often equipped with Vernier scales with which to measure h as accurately as possible. But it is also clear that it is still difficult to measure accurately elevation differences of the order of millimeters or, at the other end of the spectrum, to measure elevation differences greater than several meters. Consequently care must be given to the choice of liquid in order to produce elevation differneces that are both reasonable (of order $10 - 100cm$) and large enough to measure accurately. Thus water or alcohol are used for smaller pressure differences whereas mercury is used for larger pressure differences.

A simple manometer used in measuring the pressure difference between two tanks of liquid (rather than gas) is sketched in Figure 2. In this case the hydrostatic variation in pressure cannot be neglected and the specific pressure difference that is sought will be the difference in pressures, $(p_2 - p_1)$, at the same horizontal level as shown in the sketch. Then proceeding along the fluid path the pressure at A will be $(p_1 + \rho_1 g h_1)$ (where ρ_1 is the density of the liquid in the tanks), then the pressure at B and at C will be $(p_1 + \rho_1 g h_1 + \rho g h)$ (where ρ is the density of the denser liquid in the manometer) and thence the pressure in the right-hand tank will be given by

$$p_2 - p_1 = (\rho - \rho_1) g h \qquad \text{(Cd2)}$$

Frequently mercury is used as the denser liquid to measure pressure differences in water tanks.

If, however, this results in a elevation difference, h, that is too small to be measured accurately, an alternative arrangement is an “inverted gas/liquid manometer” which is shown schematically in Figure 3 and whose calibration is

$$p_2 - p_1 = \rho g h \qquad \text{(Cd3)}$$

where ρ is the density of the liquid in the tanks. Clearly there are other possible configurations such as an inverted alcohol/water manometer.

Another class or manometers are those designed to measure absolute pressures rather than pressure differences. The simplest barometer to measure the atmospheric pressure is sketched in Figure 4. Care

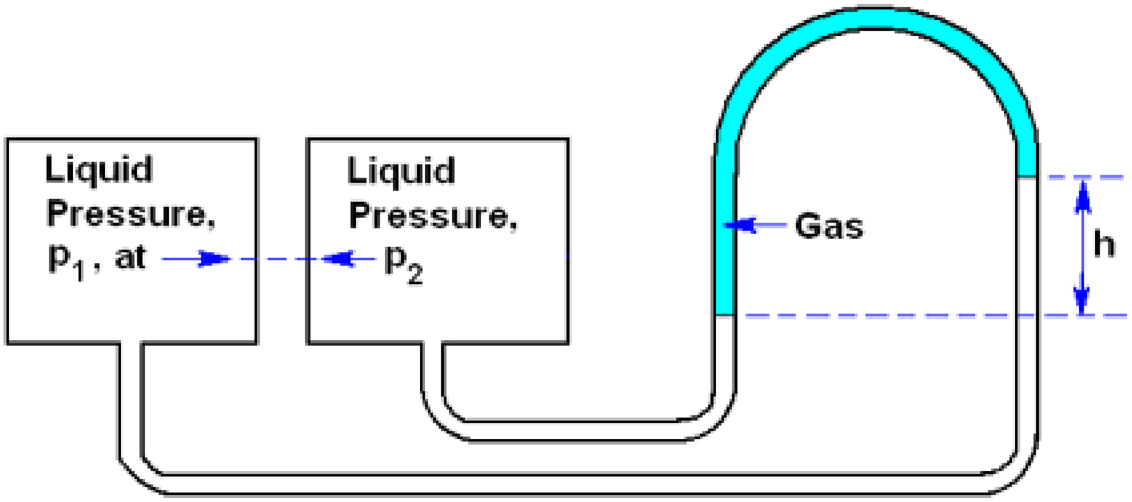

Figure 3: An inverted gas/liquid manometer.

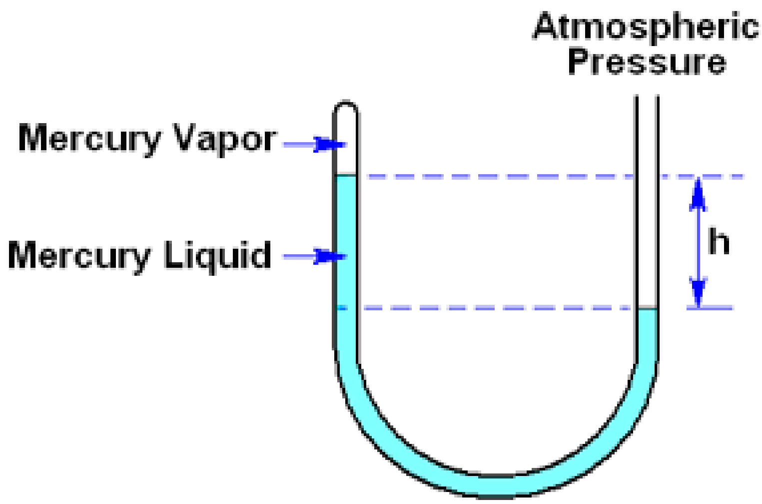

Figure 4: A mercury barometer.

is taken to completely fill the closed-topped, right-hand side of the manometer with the liquid (usually mercury) so that when the left-hand side is lowered and the pressure at the top of the right-hand side is decreased below the vapor pressure of mercury, mercury vapor is formed to take up the space above the liquid on the right. It follows that the atmospheric pressure, p_A, outside the open left-hand side is given by

$$p_A = \rho g h + p_V \tag{Cd4}$$

where ρ is the density of the mercury liquid and p_V is the vapor pressure of mercury at the ambient temperature. Looking these mercury properties up in tables and measuring h allows us to calculate the absolute value of the atmospheric pressure.