Archimedes Principle for a Floating Body

Having derived **Archimedes Principle** for a submerged body we now address the circumstance of a floating body as depicted in Figure 1. The analysis for a wholly submerged body must now be altered

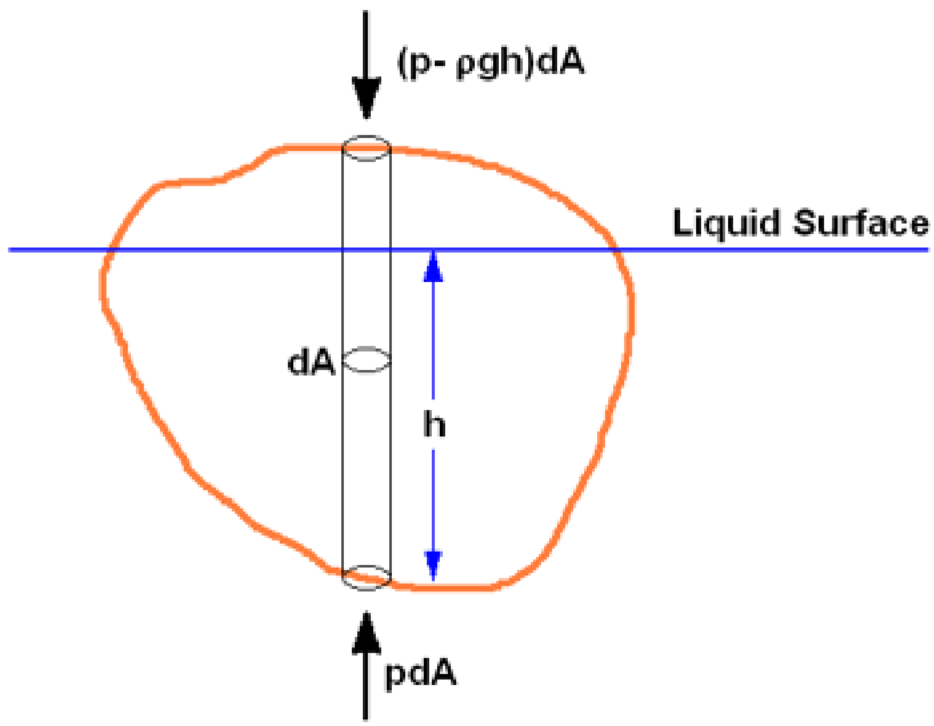

Figure 1: A floating body divided into small, vertical, cylindrical elements, dA.

since, if the pressure at the bottom end of the elemental cylinder is denoted by p, the pressure at the top end is $(p - \rho g h)$ where h is the length of the cylinder below the surrounding fluid surface rather than the total axial length of the cylinder. Consequently the net vertically-upward fluid force acting on the cylinder, dF, is again

$$dF = pdA - (p - \rho g h)dA = \rho g(hdA) = \rho g dV_F \tag{Ci1}$$

where $hdA = dV_F$ is now the volume of the cylinder below the waterline rather than the total volume of the cylinder. If we then sum up over all the cylinders that make up the body, the total vertically-upward fluid force acting on the body, F, is then

$$F = \int \rho g(hdA) = \int \rho g dV_F = \rho g V_F \tag{Ci2}$$

where V_F is the volume of the submerged body below the waterline. Since, again, $\rho g V_F$ is simply the weight of the displaced mass of fluid Archimedes principle again holds and the buoyancy force acting on a floating body is equal in magnitude to the displaced mass of fluid.

As to the horizontal force it is clear that the argument for floating bodies will be identical to that for submerged bodies and it is therefore unneccessary to repeat it.

The calculation for the line of action of this buoyancy force has the expected result that the buoyancy force acts through the center of volume of the body below the waterline. If the floating body were to tilt, this center of volume will change and therefore the **stability of floating bodies** is more complex than for wholly submerged bodies.