Pelton Wheel Turbine

The Pelton turbine (or Pelton wheel) is a particularly simple type of turbine in which a jet of water is directed at a series of *cups* or *buckets* attached to the periphery of a rotating wheel as seen in Figures 1 and 2. The momentum of the jet thus drives the wheel. The simplicity of this device means that it is and was widely used to power a multitude of different machines wherever a source of water was available. For example, the remains of Pelton turbines can often be seen in the old goldfields of the American West (see Figure 3).

Figure 1: Pelton Turbine.

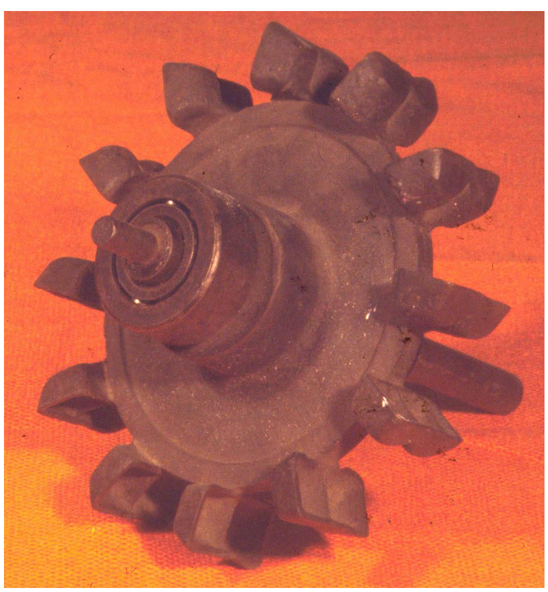

Figure 2: Pelton Wheel runner.

Figure 3: Pelton Wheels from the California goldfields.

The operating principle is sketched in Figure 4: a liquid jet of velocity, V, and cross-sectional area, A, is directed at a bucket which is thereby driven at a velocity, U, the resulting peripheral velocity of the Pelton wheel. The jet is turned by the bucket and emerges from the impact as two equal jets, each with a cross-sectional area A_2.

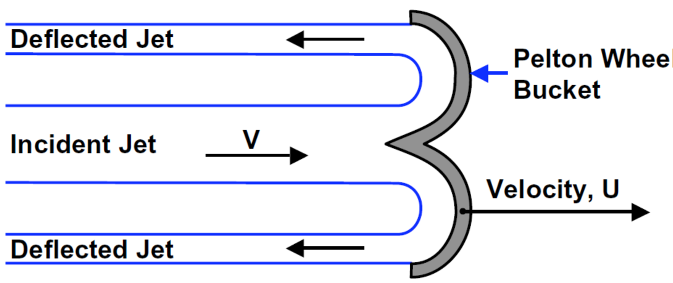

Figure 4: Pelton Turbine bucket in the non-rotating frame.

In order to apply Bernoulli's equation and the momentum theorem to the flow impinging on a bucket, it

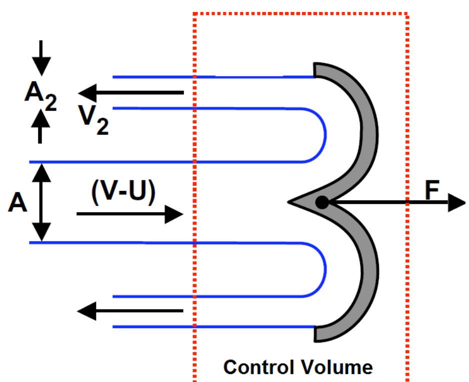

Figure 5: Pelton Turbine bucket in frame of the bucket.

is necessary to view the flow in a frame of reference in which the bucket is at rest and the flow is steady (Figure 5). In this frame the incident jet has a velocity $(V - U)$. Since both the incident and reflected jets are at the pressure of the containment vessel (see Figure 1), by Bernoulli's equation (neglecting gravity and viscous effects) they must also have the same velocity, $V_2 = V - U$, in this frame of reference. Moreover, by conservation of mass, it must also follow that $A_2 = A/2$.

Then, applying the momentum theorem to the control volume in the frame relative to the bucket (see Figure 5) it follows that the force on the bucket, F, in the direction of U is given by

$$-F = (V-U)\rho[-(V-U)]A + 2(-V_2)\rho V_2 A_2 = 2\rho A(V-U)^2 \quad \text{(Mdc1)}$$

The power, P, transmitted to the bucket and the turbine wheel is then FU where

$$P = FU = 2\rho AU(V-U)^2 \quad \text{(Mdc2)}$$

To find the velocity, U_{max}, for which the power, P, is a maximum we observe that

$$\frac{\partial P}{\partial U} = 2\rho A(V-U)^2 - 4\rho AU(V-U) \quad \text{(Mdc3)}$$

is zero when $(V-U) - 2U = 0$ and therefore

$$U_{max} = \frac{V}{3} \quad \text{(Mdc4)}$$

The ideal or theoretical hydraulic efficiency of the turbine, $\eta*$, is the ratio of the power extracted to the power available in the jet, $\rho AV^3/2$, so that

$$\eta^* = \frac{2P}{\rho AV^3} = \frac{4\rho AU(V-U)^2}{\rho AV^3} = 4\frac{U}{V}\left(1-\frac{U}{V}\right)^2 \quad \text{(Mdc5)}$$

and therefore the maximum hydraulic efficiency occurs when $U = V/3$ and is given by

$$(\eta^*)_{U/V=1/3} = \frac{4}{3}\left(\frac{2}{3}\right)^2 = \frac{16}{27} \approx 0.592 \quad \text{(Mdc6)}$$

This 59.2% maximum hydraulic efficiency is not high compared with other, more sophisticated turbines but the mechanical robustness, simplicity and ease of maintenance made the Pelton wheel optimal in many applications.

To evaluate the ideal performance that follows from the above analysis note that the head and flow coefficients, ϕ and ψ (as defined in equations (Mde7) and (Mde8)) for the Pelton wheel turbine are then given by

$$\phi = \frac{AV}{R^3\Omega} = \frac{AV}{R^2U} \quad \text{and} \quad \psi = \frac{gH}{R^2\Omega^2} = \frac{1}{2}\left(\frac{V}{U}\right)^2 \quad \text{(Mdc7)}$$

and

$$\eta* = 2\left(\frac{R^2\phi}{A\psi}\right)\left(1-\frac{A}{R^2\phi}\right)^2 \quad \text{(Mdc8)}$$

where R and Ω are the radius and radian rotational speed of the Pelton wheel.

As an example of Pelton wheel turbine performance we present data from Daugherty (1920) for a $R = 6in$ Pelton turbine operating with a variety of jet cross-sectional areas, A, non-dimensionalized as A/R^2. Figure 6 shows the head coefficient, ψ, and the actual hydraulic efficiency, η_T, plotted against the flow coefficient, ϕ for each of 7 values of A/R^2. Note that the curves for the head coefficient in Figure 6 (left) are

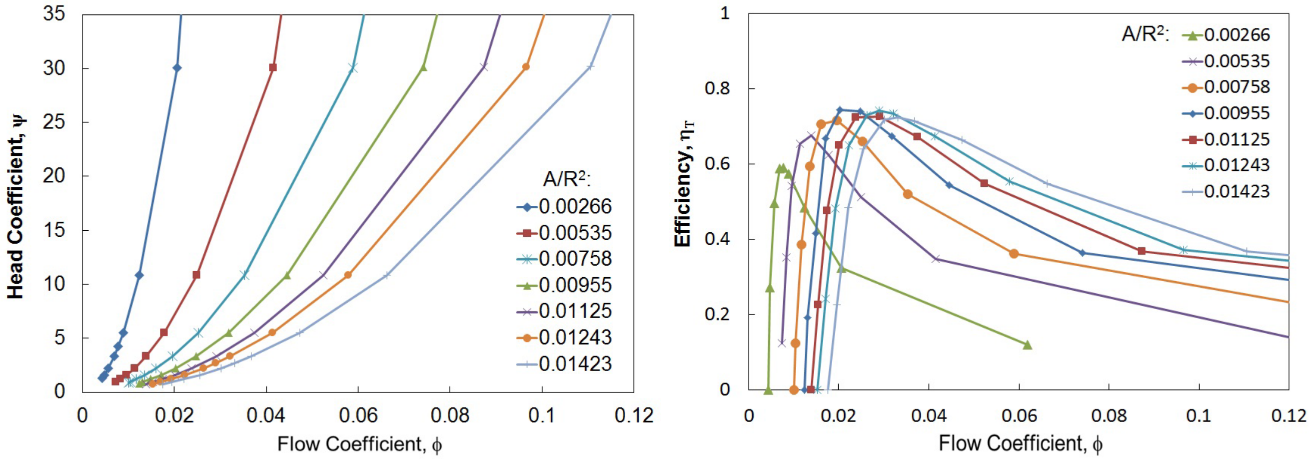

Figure 6: Head Coefficient, ψ, and efficiency, η_T, for a $R = 6in$ Pelton turbine for a series of jet areas, A/R^2 (data from Daugherty (1920)).

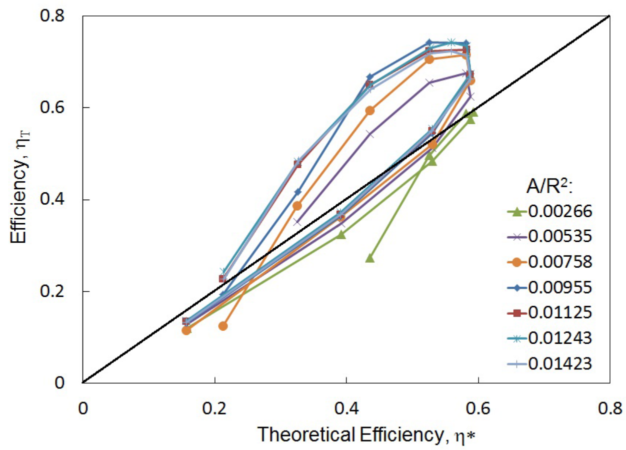

Figure 7: Comparison of the actual efficiency, η_T, in Figure 6 with the ideal, theoretical efficiency, $\eta*$, from equation (Mdc5) (data from Daugherty (1920)).

quadratics as they must be since from equations (Mdc7)

$$\psi \;=\; 0.58 \left(\frac{R^2\phi}{A}\right)^2 \qquad \text{(Mdc9)}$$

In Figure 7 the actual efficiency, η_T, is plotted against the theoretical efficiency, $\eta*$, for most of the data in Figure 6 (right) and using the values of $\eta*$ from equation (Mdc8). Not surprisingly η_T is less than $\eta*$ for most of the data (the few points below the 45° lines are for far off-design points where the data is less accurate). Around the maximum efficiency points the variation of η_T with $\eta*$ is quite consistent.