Bearings at Low Reynolds Numbers

The rotordynamics of a simple hydrodynamic bearing operating at low Reynolds number ($Re_\Omega \ll 1$)

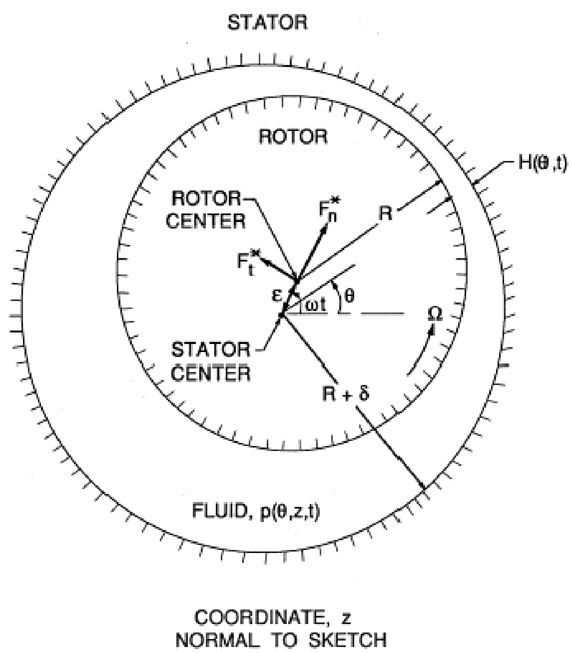

Figure 1: Schematic of fluid-filled annulus between a stator and a rotating and whirling rotor.

will be examined first. The conventional approach to this problem (Pinkus and Sternlicht 1961) is to use Reynolds' approximate equation for the fluid motions in a thin film. In the present context, in which the fluid is contained between two circular cylinders (figure 1) this equation becomes

$$\frac{1}{R^2}\frac{\partial}{\partial\theta}\left(H^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(H^3\frac{\partial p}{\partial z}\right)=6\mu\left\{2\frac{\partial H}{\partial t}+\frac{1}{R}\frac{\partial}{\partial\theta}(HU)\right\} \quad \text{(Mcd1)}$$

where (θ, z) are the circumferential and axial coordinates. This equation must be solved to find the pressure, $p(\theta, z, t)$, in the fluid (averaged over the radial extent of the clearance gap) given the clearance, $H(\theta, t)$, and the surface velocity, U, of the inner cylinder ($U = \Omega R$). An eccentricity, ϵ, at a whirl frequency of ω leads to a clearance, H, given by $\delta - \epsilon\cos(\omega t - \theta)$ and substituting for H completes the formulation of equation (Mcd1) for the pressure.

The rotordynamic forces, F_n^* and F_t^*, then follow from

$$\left\{\begin{matrix} F_n^* \\ F_t^* \end{matrix}\right\} = R\int_0^L \int_0^{2\pi} p\left\{\begin{matrix} -\cos(\omega t-\theta) \\ \sin(\omega t-\theta)\end{matrix}\right\} d\theta dz \quad \text{(Mcd2)}$$

where L is the axial length of the bearing.

Two simple asymptotic solutions are readily forthcoming for linear perturbations in which $\epsilon \ll \delta$. The first is termed the "long bearing" solution, and assumes, as discussed in the last section, that the dominant perturbations to the velocity occur in the circumferential velocities rather than the axial velocities. It follows that the second term in equation (Mcd1) can be neglected as small relative to the first term.

Neglecting, in addition, all terms quadratic or higher order in ϵ, integration of equation (Mcd1) leads to

$$p = \frac{6\mu R^2\epsilon}{\delta^3}(\Omega - 2\omega)\sin(\omega t - \theta) + \frac{12\mu R^2}{\delta^3}\frac{d\epsilon}{dt}\cos(\omega t - \theta) \tag{Mcd3}$$

and to the following rotordynamic forces:

$$F_n^* = -\frac{12\pi\mu R^3 L}{\delta^3}\frac{d\epsilon}{dt} \; ; \; F_t^* = \frac{6\pi\mu R^3 L\epsilon}{\delta^3}(\Omega - 2\omega) \tag{Mcd4}$$

In steady whirling motion, $d\epsilon/dt = 0$. The expression for F_t^* implies the following rotordynamic coefficients:

$$C^* = \frac{2k^*}{\Omega} = \frac{12\pi\mu R^3 L}{\delta^3} \tag{Mcd5}$$

and $K^* = c^* = M^* = m^* = 0$.

The second, or "short bearing", solution assumes that the dominant perturbations to the velocities occur in the axial velocities; this usually requires L/R to be less than about 0.5. Then, assuming that the pressure is measured relative to a uniform and common pressure at both ends, $z = 0$ and $z = L$, integration of equation (Mcd1) leads to

$$p = z(L - z)\left[\frac{6\mu}{\delta^3}\frac{d\epsilon}{dt}\cos(\omega t - \theta) - \frac{3\mu\epsilon}{\delta^3}(\Omega - 2\omega)\sin(\omega t - \theta)\right] \tag{Mcd6}$$

and, consequently,

$$F_n^* = -\frac{\pi\mu R L^3}{\delta^3}\frac{d\epsilon}{dt} \; ; \; F_t^* = \frac{\pi\mu R L^3\epsilon}{2\delta^3}(\Omega - 2\omega) \tag{Mcd7}$$

Therefore, in the short bearing case,

$$C^* = \frac{2k^*}{\Omega} = \frac{\pi\mu R L^3}{2\delta^3} \tag{Mcd8}$$

in contrast to the result in equation (Mcd5). Notice that, for both the long and short bearing, the value of the whirl ratio, $k^*/\Omega C^*$, is 0.5. Later, we will compare this value with that obtained for other flows and other devices.

It is particularly important to note that the tangential forces in both the long and short bearing solutions are negative for $\Omega < 2\omega$, and become positive for $\Omega > 2\omega$. This explains the phenomenon of "oil whip" in hydrodynamic bearings, first described by Newkirk and Taylor (1925). They reported that violent shaft motions occurred when the shaft speed reached a value twice the critical speed of the shaft. This phenomenon is the response of a dynamic system at its natural frequency when the exciting tangential force becomes positive, namely when $\Omega > 2\omega$ (see Hori 1959). It is of interest to note that a similar critical condition occurs for high Reynolds number flow in the film (see equations (Mce1) and (Mce2)).

The simple linear results described above can be augmented in several ways. First, similar solutions can be generated for the more general case in which the eccentricity is not necessarily small compared with the clearance. The results (Vance 1988) for the long bearing become

$$F_n^* = -\frac{12\pi\mu R^3 L}{\delta^3}\;\frac{1}{(1 - \epsilon^2/\delta^2)^{\frac{3}{2}}}\frac{d\epsilon}{dt} \tag{Mcd9}$$

$$F_t^* = \frac{6\pi\mu R^3 L(\Omega - 2\omega)}{\delta^3}\;\frac{\epsilon}{(1 - \epsilon^2/\delta^2)(2 + \epsilon^2/\delta^2)^{\frac{3}{2}}} \tag{Mcd10}$$

and, in the short bearing case,

$$F_n^* = -\frac{\pi\mu R L^3}{\delta^3}\;\frac{(1 + 2\epsilon^2/\delta^2)}{(1 - \epsilon^2/\delta^2)^{\frac{5}{2}}}\frac{d\epsilon}{dt} \tag{Mcd11}$$

$$F_t^* = \frac{\pi\mu RL^3(\Omega - 2\omega)}{2\delta^3} \frac{\epsilon}{(1-\epsilon^2/\delta^2)^{\frac{3}{2}}} \tag{Mcd12}$$

These represent perhaps the only cases in which rotordynamic forces and coefficients can be evaluated for values of the eccentricity, ϵ, comparable with the clearance, δ. The nonlinear analysis leads to rotordynamic coefficients which are functions of the eccentricity, ϵ, and the variation with ϵ/δ is presented graphically in figure 2. Note that the linear values given by equations (Mcd5) and (Mcd8) are satisfactory up to ϵ/δ of the order of 0.5.

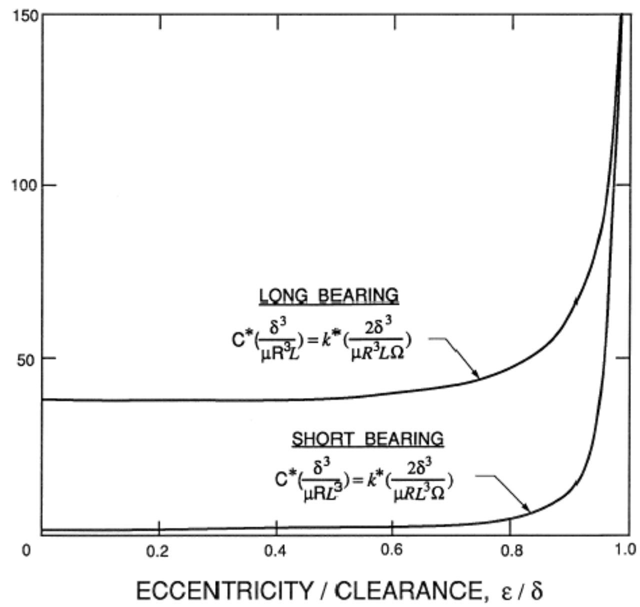

Figure 2: Dimensionless damping and cross-coupled stiffness for noncavitating long and short bearings as functions of the eccentricity ratio, ϵ/δ.

Second, it is important to note that cavitation or gas dissolution in liquid-filled bearings can often result in a substantial fraction of the annulus being filled by a gas bubble or bubbles. The reader is referred to Dowson and Taylor (1979) for a review of this complicated subject. Quite crude approximations are often introduced into lubrication analyses in order to try to account for this "cavitation". The most common approximation is to assume that the two quadrants in which the pressure falls below the mean are completely filled with gas (or vapor) rather than liquid. Called a π-film cavitated bearing, this heuristic assumption leads to the following rotordynamic forces (Vance 1988). For the cavitated (π-film) long bearing

$$F_n^* = -\frac{6\mu R^3 L}{\delta^2}\left[\frac{2|\Omega - 2\omega|\epsilon^2}{\delta^2(2+\epsilon^2/\delta^2)(1-\epsilon^2/\delta^2)} + \frac{\pi d\epsilon/dt}{\delta(1-\epsilon^2/\delta^2)^{\frac{3}{2}}}\right] \tag{Mcd13}$$

$$F_t^* = \frac{6\mu R^3 L}{\delta^3}\left[\frac{(\Omega - 2\omega)\pi\epsilon}{(2+\epsilon^2/\delta^2)(1-\epsilon^2/\delta^2)^{\frac{3}{2}}} + \frac{4d\epsilon/dt}{(1+\epsilon/\delta)(1-\epsilon^2/\delta^2)}\right] + 2RLp_0 \tag{Mcd14}$$

where p_0 is the pressure in the cavity. These expressions are similar to, but not identical with, the expressions derived by Hori (1959) and used to explain oil whip. For the cavitated (π-film) short bearing

$$F_n^* = -\frac{\mu RL^3}{\delta^2}\left[\frac{|\Omega - 2\omega|\epsilon^2}{\delta^2(1-\epsilon^2/\delta^2)^2} + \frac{\pi(1+2\epsilon^2/\delta^2)d\epsilon/dt}{2\delta(1-\epsilon^2/\delta^2)^{\frac{5}{2}}}\right] \tag{Mcd15}$$

$$F_t^* = \frac{\mu R L^3}{\delta^2} \left[\frac{(\Omega - 2\omega)\pi\epsilon}{4\delta(1-\epsilon^2/\delta^2)^{\frac{3}{2}}} + \frac{2\epsilon d\epsilon/dt}{\delta^2(1-\epsilon^2/\delta^2)^2} \right] + 2RLp_0 \tag{Mcd16}$$

It would, however, be appropriate to observe that rotordynamic coefficients under cavitating conditions remain to be measured experimentally, and until such tests are performed the above results should be regarded with some scepticism.

Finally, we note that all of the fluid inertial effects have been neglected in the above analyses, and, consequently, the question arises as to how the results might change when the Reynolds number, Re_Ω, is no longer negligibly small. Such analyses require a return to the full Navier-Stokes equations, and the author has explored the solutions of these equations in the case of long bearings (Brennen 1976). In the case of whirl with constant eccentricity ($d\epsilon/dt = 0$), it was shown that there are two separate sets of asymptotic results for $Re_\omega \ll \delta^3/R^3$, and for $\delta^3/R^3 \ll Re_\omega \ll \delta^2/R^2$. For $Re_\omega \ll \delta^3/R^3$, the rotordynamic forces are

$$F_n = -\frac{9}{4}\frac{R^5}{\delta^5}\left(1 - \frac{2\omega}{\Omega}\right) \tag{Mcd17}$$

while F_t^* is the same as given in equation (Mcd4). Notice that equation (Mcd17) implies a direct stiffness, K, and cross-coupled damping, c, given by

$$K = \frac{c}{2} = \frac{9}{4}\frac{R^5}{\delta^5} \tag{Mcd18}$$

On the other hand, for the range of Reynolds numbers given by $\delta^3/R^3 \ll Re_\omega \ll \delta^2/R^2$, the rotordynamic forces are

$$F_n = \frac{16\delta}{R\ Re_\Omega^2}\left(\frac{2\omega}{\Omega} - 1\right) \quad ; \quad F_t = \frac{128\delta^4}{3R^4 Re_\Omega^3}\left(1 - \frac{2\omega}{\Omega}\right) \tag{Mcd19}$$

so that

$$K = \frac{c}{2} = \frac{16\delta}{R\ Re_\Omega^2} \quad ; \quad C = 2k = \frac{256\delta^4}{3R^4 Re_\Omega^3} \tag{Mcd20}$$

In both cases the direct stiffness, K, is positive, implying a positive hydrodynamic restoring force caused by the inertial terms in the equations of fluid motion.