Free Streamline Methods

The diversity of types of cavitation in a pump and the complexity of the two-phase flow which it generates mean that reliable analytical methods for predicting the cavitating performance characteristics are virtually non-existent. However if the cavity flow can be approximated by single, fully developed or attached cavities on each blade, then this allows recourse to the methods of free streamline theory for which the reader may wish to consult the reviews by Tulin (1964) and Wu (1972) or the books by Birkhoff and Zarantonello (1957) and Brennen (1994). The analytical approaches can be subdivided into linear theories which are applicable to slender, streamlined flows (Tulin 1964) and non-linear theories which are more accurate but can be mathematically much more complex (Wu 1972). Both approaches to free streamline flows have been used in a wide range of cavity flow problems and it is necessary to restrict the present discussion to some of the solutions of relevance to attached cavitation in pumps.

It is instructive to begin by quoting some of the results obtained for single hydrofoils for which the review by Acosta (1973) provides an excellent background. In particular we will focus on the results of approximate linear theories for a partially cavitating or supercavitating flat plate hydrofoil. The partially cavitating solution (Acosta 1955) yields a lift coefficient

$$C_L = \pi\alpha\left[1 + (1-\ell)^{-\frac{1}{2}}\right] \tag{Mbes1}$$

where ℓ is the ratio of the cavity length to the chord of the foil and is related to the cavitation number, σ, by

$$\frac{\sigma}{2\alpha} = \frac{2-\ell+2(1-\ell)^{\frac{1}{2}}}{\ell^{\frac{1}{2}}(1-\ell)^{\frac{1}{2}}} \tag{Mbes2}$$

Thus, for a given cavity length, ℓ, and a given angle of incidence, α, the cavitation number follows from equation (Mbes2) and the lift coefficient from equation (Mbes1). Note that as $\ell \to 0$ the value of C_L tends to the theoretical value for a non-cavitating flat plate, namely $2\pi\alpha$. The corresponding solution for a supercavitating flat plate was given by Tulin (1953) in his pioneering paper on linearized cavity flows. In this case

$$C_L = \pi\alpha\ell\left[\ell^{\frac{1}{2}}(\ell-1)^{-\frac{1}{2}} - 1\right] \tag{Mbes3}$$

$$\alpha\left(\frac{2}{\sigma}+1\right) = (\ell-1)^{\frac{1}{2}} \tag{Mbes4}$$

where now, of course, $\ell > 1$.

The lift coefficient and the cavity length from equations (Mbes1) to (Mbes4) are plotted against cavitation number in figure 1 for a typical angle of incidence of $\alpha = 4°$. Note that as $\sigma \to \infty$ the fully wetted lift coefficient, namely $2\pi\alpha$, is recovered from the partial cavitation solution and that as $\sigma \to 0$ the lift coefficient tends to $\pi\alpha/2$. Notice also that both the solutions become pathological when the length of the cavity approaches the chord length ($\ell \to 1$). However, if some small portion of each curve close to $\ell = 1$ were eliminated, then the characteristic decline in the performance of the hydrofoil as the cavitation number is decreased is readily observed. It also compares well with the experimental observations as illustrated by the favorable comparison with the data of Wade and Acosta (1966) included in figure 1. Consequently, as the cavitation number is decreased, a single foil exhibits only a small change in the performance or lift coefficient until some critical value of σ (about 0.7 in the case of figure 1) is reached. Below this critical value the performance begins to "breakdown" quite rapidly. Thus, even a single foil mirrors the typical cavitation performance experienced in a pump. On a more detailed level, note that the small increase in the supercavitating lift coefficient which occurs as the cavitation number is decreased toward the critical

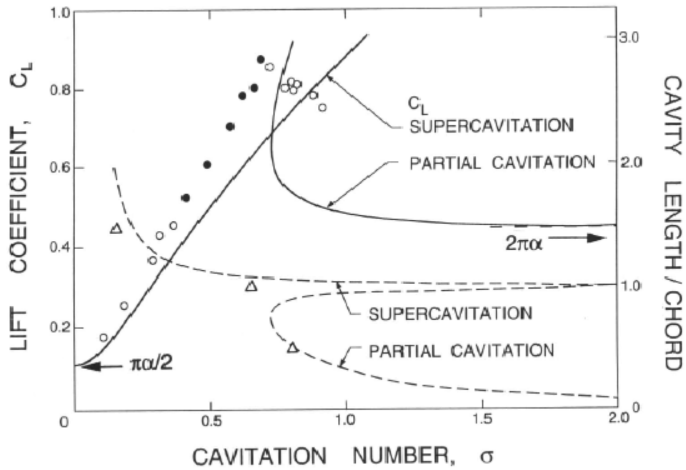

Figure 1: Typical results from the linearized theories for a cavitating flat plate at an angle of incidence of 4°. The lift coefficients, C_L (solid lines), and the ratios of cavity length to chord, ℓ (dashed lines), are from the supercavitation theory of Tulin (1953) and the partial cavitation theory of Acosta (1955). Also shown are the experimental results of Wade and Acosta (1966) for ℓ ($\triangle$) and for C_L ($\circ$ and $\bullet$) where the open symbols represent points of stable operation and the solid symbols denote points of unstable cavity operation.

value of σ is, in fact, observed experimentally with many single hydrofoils (for example, Wade and Acosta 1966) as well as in some pumps.

The peculiar behaviour of the analytical solutions close to the critical cavitation number is related to an instability which is observed when the cavity length is of the same order as the chord of the foil. However, we delay further discussion of this until the appropriate point in the next chapter (see section (Mbfj)). Some additional data on the variation of the lift coefficient with angle of incidence is included in that later section.

Before leaving the subject of the single cavitating foil we should note that more exact, non-linear solutions for a flat plate or an arbitrarily shaped profile have been generated by Wu (1956, 1962), Mimura (1958) and others. As an example of these non-linear results, the lift and drag coefficients at various cavitation numbers and angles of incidence are presented in figures 2 and 3 where they are compared with the experimental data of Parkin (1958) and Silberman (1959). Data both for supercavitating and partially cavitating conditions are shown in these figures, the latter occurring at the higher cavitation numbers and lower incidence angles (the dashed parts of the curves represent a somewhat arbitrary smoothing through the critical region in which the cavity lengths are close to the chord length). This comparison demonstrates that the non-linear theory yields values which are in good agreement with the experimental measurements. In the case of circular-arc hydrofoils, Wu and Wang (1964) have shown similar agreement with the data of Parkin (1958) for this type of profile. For a recent treatment of supercavitating single foils the reader is referred to the work of Furuya and Acosta (1973).

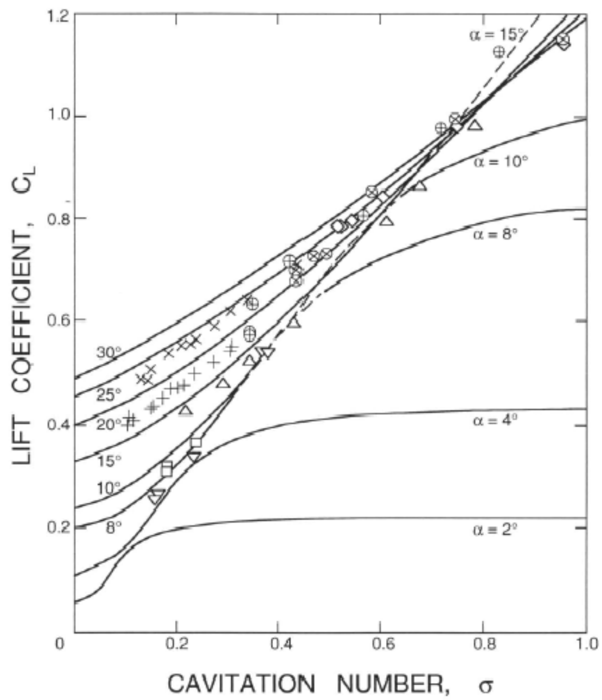

Figure 2: Lift coefficients for a flat plate from the non-linear theory of Wu (1962). The experimental data (Parkin 1958) is for angles of incidence as follows: 8° (▽), 10° (□), 15° (△), 20° (⊕), 25° (⊗), and 30° (◇). Also shown is some data of Silberman (1959) in a free jet tunnel: 20° (+) and 25° (×).

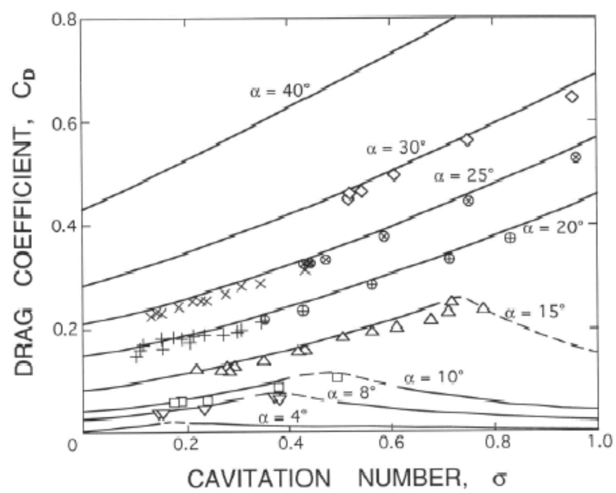

Figure 3: Drag coefficients corresponding to the lift coefficients of figure 2.