Idealized Non-cavitating Pump Performance

It is useful at this point to develop an approximate and idealized evaluation of the hydraulic performance of a pump in the absence of cavitation. This will take the form of an analytical expression for the head rise (or ψ) as a function of the flow rate (or ϕ_2).

To simplify this analysis it is assumed that the flow is incompressible, axisymmetric and steady in the rotating framework of the impeller blades; that the blades are infinitely thin; and that viscous losses can be neglected. Under these conditions the flow in any streamtube, such as depicted in figure 2, section (Mbba), will follow the Bernoulli equation for a rotating system (see, for example, Sabersky, Acosta and Hauptmann 1989),

$$\frac{2p_1}{\rho} + w_1^2 - r_1^2\Omega^2 = \frac{2p_2}{\rho} + w_2^2 - r_2^2\Omega^2 \qquad \text{(Mbbg1)}$$

This equation can be usefully interpreted as an energy equation as follows. The terms $p + \frac{1}{2}\rho w^2$ on either side are the total pressure or mechanical energy per unit volume of fluid, and this quantity would be the same at inlet and discharge were it not for the fact that "potential" energy is stored in the rotating fluid. The term $\rho(r_1^2 - r_2^2)\Omega^2/2$ represents the difference in this "potential" energy at inlet and discharge. Clearly, when there are losses, equation (Mbbg1) will no longer be true.

Using the definition of the total pressure (equation (Mbbc1)) and the relations between the velocities derived from the velocity triangles of figure 2, section (Mbba), equation (Mbbg1) can be manipulated to yield the following expression for the total pressure rise, $(p_2^T - p_1^T)$, for a given streamtube:

$$p_2^T - p_1^T = p_2 - p_1 + \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \qquad \text{(Mbbg2)}$$

$$p_2^T - p_1^T = \rho(\Omega r_2 v_{\theta 2} - \Omega r_1 v_{\theta 1}) \qquad \text{(Mbbg3)}$$

In the absence of inlet swirl ($v_{\theta 1} = 0$), this leads to the nondimensional performance characteristic

$$\psi = 1 - \phi_2 \cot \beta_{b2} \qquad \text{(Mbbg4)}$$

using the definitions in equations (Mbbc4) and (Mbbc5). Here we have assumed that the inlet and discharge conditions are uniform which, in effect, restricts the result to a turbomachine in which the widths, B_1 and B_2 (figure 1, section (Mbba)), are such that $B_1 \ll R_{T1}, B_2 \ll R_{T2}$, and in which the velocities of the flow and the impeller are uniform across both the inlet and the discharge. Usually this is not the case, and the results given by equations (Mbbg3) and (Mbbg4) then become applicable to each individual streamtube. Integration over all the streamtubes is necessary to obtain the performance characteristic for the machine. An example of this integration was given in section (Mbbc). Even in these nonuniform cases, the simple expression (Mbbg4) is widely used in combination with some mean or effective discharge blade angle, β_{b2}, to estimate the performance of a pump.

It is important to note that the above results can be connected with those of the preceding section by applying the angular momentum theorem (Newton's second law of motion applied to rotational motion) to relate the torque, T, to the net flux of angular momentum out of the pump:

$$T = m(r_2 v_{\theta 2} - r_1 v_{\theta 1}) \qquad \text{(Mbbg5)}$$

where, as before, m is the mass flow rate. Note that this momentum equation (Mbbg5) holds whether or not there are viscous losses. In the absence of viscous losses, a second expression for the torque, T, follows from equation (Mbbf5). By equating the two expressions, the result (Mbbg3) for the performance in the absence of viscous losses is obtained by an alternative method.