Rocket Propulsion

A modern rocket consists of a payload of very small mass, M_P, mounted on a much larger mass of fuel/oxidizer, $M_F(0)$, that is burnt in a combustion chamber at high pressure and exhausted through a supersonic nozzle to produce a very high speed exhaust jet of velocity, U_j. A modern rocket has one of two basic propulsion systems, either

- it is propelled by a liquid fuel stored in a large tank and burnt in a combustion chamber using a liquid oxidizer stored in a second large tank as sketched in Figure 1
- or it is propelled by a solid that is a combination of fuel and oxidizer that is burnt in place as sketched in Figure 2.

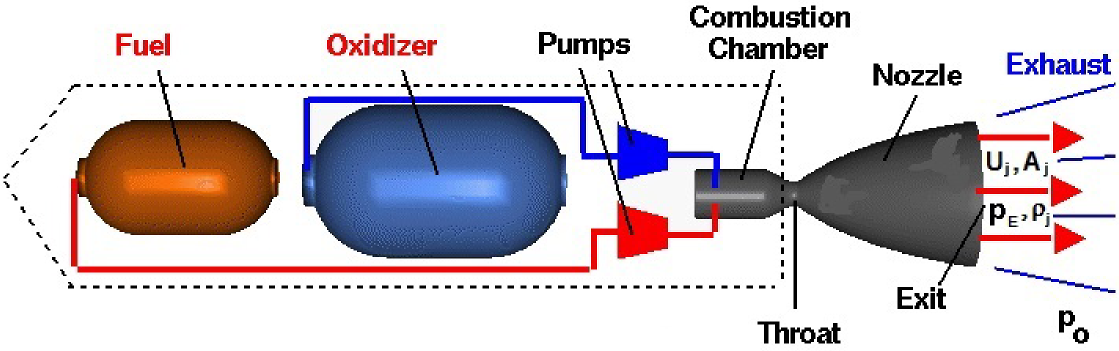

Figure 1: Schematic of a liquid-propelled rocket.

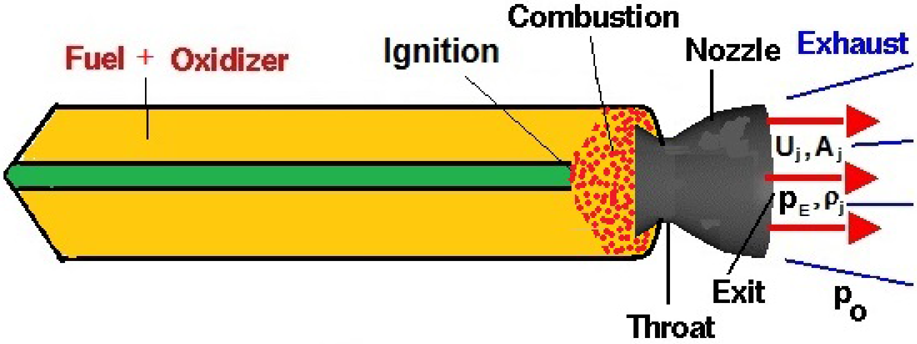

Figure 2: Schematic of a solid-propelled rocket or booster.

In either case the high temperature combustion products are accelerated through a supersonic nozzle that creates a very high speed supersonic jet that generates the rocket thrust.

The thrust, T, is most readily understood through the application of the linear momentum theorem. Consider the sketches of rockets shown in Figures 1 and 2. The mass flow rate out of the rocket is denoted by $\dot{M}$ where

$$\dot{M} = \rho_j U_j A_j \tag{Ddf1}$$

where ρ_j, U_j and A_j are the density, velocity and cross-sectional area of the rocket engine exhaust. Consequently, by Newton's law, the momentum flux in the exhaust is $\rho_j U_j^2 A_j$ and the thrust produced by the engine which propels the rocket, T, is

$$T \;=\; \dot{M} U_j + A_j(p_E - p_0) \;=\; \rho_j U_j^2 A_j + A_j(p_E - p_0) \tag{Ddf2}$$

where p_E is the pressure of the rocket engine exhaust and p_0 is the ambient pressure surrounding the jet and the vehicle. Sometimes, the two terms on the right-hand side are combined by defining an equivalent exhaust velocity, U_j^*, such that

$$T \;=\; \dot{M} U_j^* \tag{Ddf3}$$

In the above simplified analysis we have assumed that the velocity, U_j, is uniform across the jet area, A_j. As in the case of the jet engine non-uniformity increases the thrust for a given flow rate, $\dot{M}$.

The *total impulse*, I, of a rocket is defined as the thrust, T, integrated over the total time of firing, t_T:

$$I \;=\; \int_0^{t_T} T \; dt \tag{Ddf4}$$

and substituting for T from equation (Ddf3)

$$I \;=\; \int_0^{t_T} U_j^* \dot{M} \; dt \tag{Ddf5}$$

which, if the exhaust velocity is approximately constant during the flight, yields

$$I \;=\; M_F(0) U_j^* \tag{Ddf6}$$

where $M_F(0)$ is the total mass of fuel in the rocket. It is conventional to define a quantity, I_S, known as the *specific impulse* as

$$I_S \;=\; \frac{I}{M_F(0)} \;=\; U_j^* \tag{Ddf7}$$

Thus the specific impulse is approximately equal to the exhaust velocity, U_j. The specific impulse is a useful and widely used measure of the "efficiency" of a rocket engine. In the above definition the units of the specific impulse are the units of velocity. However, it is quite common to divide I by the total weight of fuel rather than the total mass of fuel and the resulting units are then the units of time. It is usually quoted in seconds.

The thrust, T, accelerates the rocket usually to orbital velocities. As the rocket velocity, $V(t)$ increases with accelerations, $dV(t)/dt$ (of the order of $3g$), the mass of the fuel, $M_F(t)$, is decreasing so that, if we neglect the drag on the rocket caused by the flow around it,

$$T \;=\; \dot{M} U_j^* \;=\; -U_j^* \frac{dM_F(t)}{dt} \;=\; \frac{d}{dt}\left\{(M_P + M_F(t))V(t)\right\} \tag{Ddf8}$$

This represents an ordinary differential equation that must be solved to find the rocket velocity, $V(t)$, given the initial masses, the jet velocity, U_j, and the pressure difference, $(p_E - p_0)$. When this pressure difference is small equation (Ddf8) becomes

$$-U_j \frac{dM_F(t)}{dt} \;=\; \frac{d}{dt}\left\{(M_P + M_F(t))V(t)\right\} \tag{Ddf9}$$

and an equation like equation (Ddf8) or (Ddf9) must be solved to determine the progress of the rocket, $V(t)$.

Table 1: Data for some liquid- and solid-propelled rocket engines. The mixture ratio is the ratio of oxidizer to fuel.

LIQUID: Fuel/Oxidizer	Rocket, Stage	Engine	Mixture Ratio (%)	Specific Impulse in vacuum (s)
RP-1/LOX	Atlas/Centaur, Stage 1	Rocketdyne YLR105-NA7	2.29	309
RP-1/LOX	Saturn V, Stage 1	Rocketdyne F-1	2.29	304
RP-1/LOX	Delta II, Stage 1	Rocketdyne RS-27	2.29	295
LH2/LOX	Atlas/Centaur, Stage 2	Pratt & Whitney RL-10A	5.00	444
LH2/LOX	Saturn V, Stage 3	Rocketdyne J-2	5.00	424
LH2/LOX	Space Shuttle Main Engine	Rocketdyne	5.00	453
Aerozine/N2O	Titan II, Stage 2	Aerojet LR-91-AJ	1.59	312
Aerozine/N2O	Delta II, Stage 2	Aerojet AJ10-118K	1.59	320

SOLID: Fuel	Rocket, Stage	Engine	Specific Impulse in vacuum (s)
PBAN Solid	Space Shuttle Booster	Thiokol SRB	268
HTPB Solid	Delta II Booster	Castor 4A	266

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It transpires that the specific impulse, I_S, is primarily determined by the properties of the fuel and oxidant used in the rocket. Table 1 lists a number of liquid- and solid-propellant rockets, the fuels and oxidants used (see below) and the specific impulse of those rockets when exhausting to vacuum. With liquid-propelled rockets it transpires that the liquid hydrogen (LH2)/ liquid oxygen (LOX) combination produces a specific impulse of approximately $450s$, substantially higher than all the other realistic combinations. By comparison, as shown in Table 1, modern solid rockets have specific impulses of about $250s$.

The choice of a propulsion system and fuel for a rocket is not only based on the specific impulse but also on a number of other factors:

- A solid propellant is easier to store and the rocket is simpler and cheaper to manufacture.
- However it is unrealistic to try to control or limit the burn of a solid-propelled rocket; once it is ignited it burns out completely.
- A liquid-propelled rocket can be controlled and can be stopped and restarted, capabilities that are necessary for controlled placement of many payloads. Thus the Space Shuttle, like many modern rockets, utilized a combination of solids for the heavy lifting and liquids for control.
- Liquid-propelled rockets are not only much more expensive to manufacture but also require careful management in the loading of the fuel and oxidizer prior to launch. When a rocket is designed as a weapon requiring rapid deployment at short notice, the otherwise preferable fuel and oxidizer combination of LH2 (liquid hydrogen) and LOX (liquid oxygen) has severe disadvantages and consequently various storable liquid fuel/oxidizer combinations such a hydrazine/nitrogen tetroxide were devised and deployed.