Finite Span Performance

In the preceding section (Dce) the performance of a two-dimensional airfoil was described and discussed. Those discussions were pertinent, for example, to planar flows around airfoils like those installed in and spanning a rectangular wind tunnel. However, in many applications, for example as aircraft wings or propeller blades, airfoils have a finite span and the performance of those finite-span wings or blades may differ significantly from the performance of the corresponding two-dimensional airfoils. In this section we address those differences.

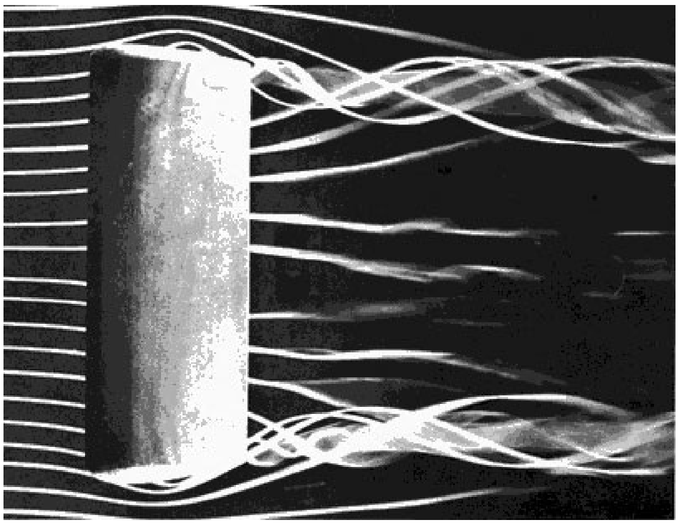

Figure 1: Tip vortices in the wake of an airfoil at a Reynolds number of 10^5. From Head (1982).

The primary reason for these differences is that the flow curls round the free end of the wing or blade thus decreasing the pressure difference between the upper, suction surface of the foil and the lower pressure surface. Thus the lift produced by the foil is reduced. This curling flow around the end produces the

Figure 2: The wing tip vortices on a EA-6B Prowler and in the wake of an airliner.

trailing vortices (see, for example, Figures 1) that are a fundamental component of the wake of an aircraft as illustrated by the photographs in Figure 2. The following videos taken by Marc Grossman provide a nice illustration of wingtip vortices and their break-up into "vortex rings": https://youtu.be/fCbK-LEUkOY, https://youtu.be/wR8PIIxROb4.

Clearly, the effect of finite span will depend on the planform geometry of the airfoil, wing or blade. The span is clearly definable as the tip-to-tip distance along the surface of the wing(s) or the root-to-tip length of a blade and this is denoted by s. The chord, c, is traditionally the distance from the leading edge to the trailing edge measured in the direction of the oncoming flow. The aspect ratio of the airfoil could then be envisaged as the ratio, s/c, so that the two-dimensional performance data of the last section (Dce) should be labeled the infinite aspect ratio performance. However, c, will usually vary considerably with spanwise position and so it is more useful to define the aspect ratio of the foil, Ar, as s^2/A_P where A_P is the planform area of the wing or blade (clearly Ar is equal to the ratio of the span to the chord for a rectangular planform). It is self-evident that the performance of a foil will decrease as the aspect ratio decreases; in particular, the maximum L/D will decrease as Ar decreases. For reference some typical aspect ratios and L/D values are listed in Table 1.

Table 1: Table 1. Typical Ar and L/D values for some common foils.

Foil:	Ar	L/D
Modern sailplane	30	40 – 60
Hang glider	6	15
Albatross	12 – 15	20
Boeing 767	8	17
Sparrow	??	4
Light Plane (Cessna 150)	6 – 8	7
Sail Boat Sail	10	10
Wingsuit	0.7	2.5
Swim Fins	0.3 – 0.5	2

Clearly and understandably the higher the aspect ratio the better the airfoil performance. However there are design factors that effect the aspect ratio such as the required structural strength of the wing or blade. Thus, for example, ship propeller blades necessarily have a much lower aspect ratio than airplane propellers because they must withstand much larger forces.

Figure 3 shows the effect of a finite aspect ratio on the lift and the drag. As seen on the left in that figure

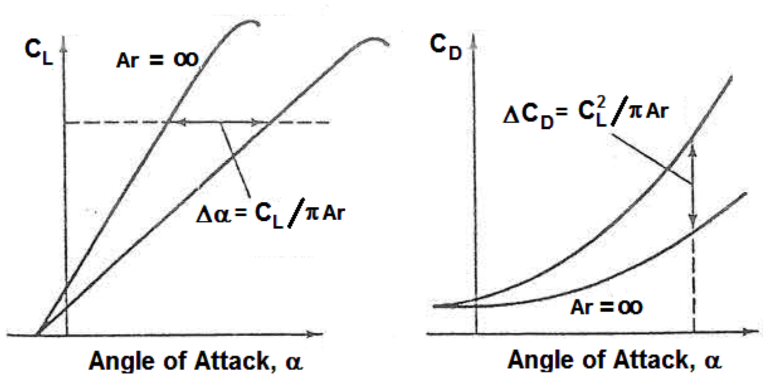

Figure 3: The effect of finite aspect ratio on the lift and drag of a typical airfoil. Adapted from White (1999).

Figure 4: Typical high-aspect ratio glider.

the angle of attack required to produce the same C_L as the infinite span foil is greater than that of the infinite span foil by an amount equal to $C_L/\pi Ar$. Moreover as shown on the right in Figure 3 the drag coefficient for the finite aspect foil is increased by $C_L^2/\pi Ar$. Thus, as the aspect ratio is decreased, the lift decreases, the drag increases and the lift-drag ratio substantially decreases. Therefore airfoil designers strive for the largest practical aspect ratio in order to optimize the fluid dynamic performance of the foil. Thus, for example, lightweight gliders like that of Figure 4 have very high aspect ratios because the lift they must produce is very small and therefore the structural strength required of the wings is quite limited. Larger, heavier aircraft with greater wing stresses require smaller aspect ratio wings. Aircraft propellers (and turbine blades) can have much larger aspect ratios than marine propellers or water turbines for the same reason. Therefore the curling up of the flow at the wing tips and the trailing vortices this produces are inescapable features of the fluid dynamics of finite-span airfoils (or hydrofoils). Sometimes attachments to the wing tips are deployed in an attempt to mitigate the adverse effects but the added drag that these attachments produce often makes them less effective than might have been envisaged.

Another consequence of the wing tip vortices is that they induce a downward component of velocity in the wake between the two tip vortices as can be visualized in the photographs, Figures 1 and 2. This is called the *downwash* and the component of velocity is called the *downwash velocity* (Figure 5). This velocity will

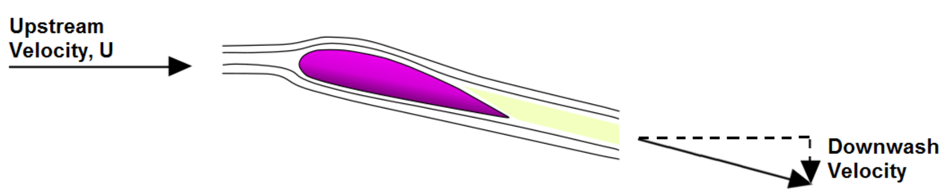

Figure 5: Longitudinal plane cut through the flow past a finite-aspect ratio airfoil.

be proportional to the lift for without any lift there would be no tip vortices and no downwash. Since the lift is proportional to the angle of attack, α, it follows that the downwash velocity is also proportional to α. Moreover, the downwash is zero for an infinite span and therefore we might suspect that the downwash velocity will also be inversely proportional to the aspect ratio, Ar, so that the downwash velocity $\propto \alpha/Ar$. Furthermore the lift force acting on this section of the foil will no longer be perpendicular to the oncoming stream but inclined at some angle to that direction, an angle that will be proportional to the downwash velocity even in inviscid flow. Therefore the forces on this section of the airfoil and consequently on the airfoil as a whole (even in inviscid, potential flow) will not be perpendicular to the oncoming stream (as they are for the two-dimensional airfoil in inviscid potential flow) but inclined backward at some small angle proportional to the downwash velocity. The component of this force in the direction of the oncoming stream represents a drag force that is called the *induced drag*; this is a rare circumstance in which there is a drag force in an inviscid potential flow. The work that the foil does in overcoming this induced drag goes into energy in the tip vortices.

It follows from the above that the induced drag will be roughly proportional to $L\alpha/Ar$ and the additional contribution to the drag coefficient, ΔC_D will be proportional to $C_L\alpha/Ar$. Since $C_L \approx 2\pi\alpha$ it follows that

$$\Delta C_D \propto \frac{C_L \alpha}{Ar} \qquad \text{or} \qquad \Delta C_D \propto \frac{C_L^2}{2\pi Ar} \tag{Dcf1}$$

A more quantitative analysis of this three-dimensional flow (see White (1999)) can yield the factor of proportionality in this last relation and it transpires that this factor has a value of about 2 so that $\Delta C_D \approx C_L^2/\pi Ar$. This is the origin of the finite aspect ratio correction discussed above and depicted in Figure 3.