Introduction to External Flows

In this and the sections which follow we focus attention on external flows around objects and the forces, moments and other effects which the object experiences as a result. First we examine the general nature of the forces that the flow imposes on the object since effects like the drag and lift play a substantial role in our everyday lives. To simplify our initial discussions we limit attention to the forces generated by a steady, uniform flow (velocity, U) and define Cartesian axes such that x is in the same direction as that uniform stream. In the case of planar flows around two-dimensional objects the y axis is in the plane of the flow and the z axis is in the third perpendicular direction. Then the *drag*, D, is the force on the object in the x direction and the lift forces, L, are the forces in the y and z directions.

We begin by using dimensional analysis to establish the parameters that the drag and lift will depend upon. To do so it is convenient to define non-dimensional drag and lift coefficients, C_D and C_L, which are conventionally defined as

$$C_D \;=\; \frac{D}{\frac{1}{2}\rho U^2 A^*} \quad \text{and} \quad C_L \;=\; \frac{L}{\frac{1}{2}\rho U^2 A^*} \qquad \text{(Daa1)}$$

where ρ is the density of the fluid at some specific reference point in the flow and A^* is a cross-sectional area that may differ according to the type of flow being considered and must therefore be carefully prescribed in each case. In dealing with bluff bodies of arbitrary shape it is conventional to use the frontal projected area (the area of the object when viewed from far upstream) for A^*. On the other hand, in dealing with airfoils it is conventional to use the planform area of the foil for A^* so that A^* does not change as the angle of attack on the foil changes.

With these definitions established we proceed to consider the fluid properties and flow characteristics upon which C_D and C_L might depend. The fluid forces on an object in a flow must depend on the flow characteristics, the geometry and size of the object and the properties of the fluid. Consequently with can write

$$D \quad \text{or} \quad L \;=\; \text{function of}\,\{U, \ell, \text{Object Shape and Orientation}, \rho, \mu, S, \theta, g, c\} \qquad \text{(Daa2)}$$

where U is the velocity of the oncoming stream, ℓ is a single dimension indicating the size of the object, the object shape includes all the dimensionless ratios needed to define the shape and orientation of the object, ρ and μ (or ν) are the density and viscosity of the fluid, S and θ are the surface tension and contact angle of the fluid in contact with the object, g is the gravitational acceleration and c is the speed of sound in the fluid. This list is necessarily incomplete but will serve our present purposes. Non-dimensionalizing equation (Daa2) we can write

$$C_D \quad \text{or} \quad C_L \;=\; \text{function of} \left\{ \frac{\rho U \ell}{\mu},\; \frac{U}{(g\ell)^{\frac{1}{2}}} \;,\; \frac{U}{c} \;,\; \text{Object Shape and Orientation} \right\} \qquad \text{(Daa3)}$$

where we have omitted the surface tension and contact angle effects for simplicity. Moreover on the right-hand side we recognize the Reynolds number, $Re = \rho U \ell/\mu$, the Froude number $Fr = U/(g\ell)^{\frac{1}{2}}$ and the Mach number, $M = U/c$ so that

$$C_D \quad \text{or} \quad C_L \;=\; \text{function of}\,\{\; Re \;,\; Fr \;,\; M \;,\; \text{Object Shape and Orientation} \;\} \qquad \text{(Daa4)}$$

The Froude number will only be a factor when there is a liquid/gas interface near the object that effects the flow (as, for example, a surface ship) and the Mach number only becomes a factor when it is of order

0.25 or greater. Thus, in flows without a free surface that are essentially incompressible (say $M < 0.25$) the relation (Daa4) reduces to

$$C_D \quad \text{or} \quad C_L \;=\; \text{function of}\,\{\; Re \;,\; \text{Object Shape and Orientation} \;\} \tag{Daa5}$$

and we will initially focus on such flows in order to establish a baseline data set. We will examine the dependence of C_D and C_L on the object shape and orientation and on the Reynolds number, Re, in the sections which follow. The effects of Froude number, Fr, and Mach number, M, will also be examined in the relevant contexts.