Forces on Objects in a Flow

The forces on an object in a flow can be decomposed into those forces caused by the normal stresses or pressures acting on the surface of the object and forces caused by the shear stresses acting on the surface. The forces caused by the pressure are called the *form forces*, *Form Drag* and *Form Lift* (though the latter term is rarely used), while the forces caused by the shear stresses are called the *skin friction forces*, *Skin Friction Drag* and *Skin Friction Lift* (though, again, the latter terminology is rarely used). It is appropriate to consider the relations between these various forces and previously defined local parameters such as the coefficient of pressure.

Consider first the form drag in the steady flow of a uniform velocity, U, past a finite object as depicted in figure 1. Dividing up the volume of the object into a series of cylinders of cross-sectional area, dA, parallel with the oncoming stream and the x axis and denoting the pressures on the forward-facing and rearward-facing surfaces by p_1 and p_2 respectively it follows that the form drag on the object, D_F, will be

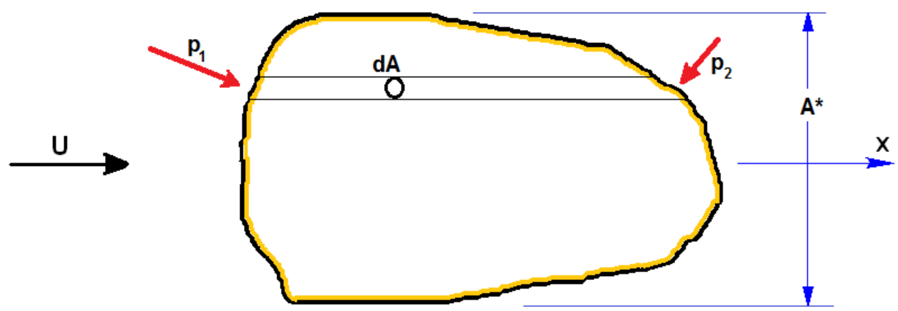

Figure 1: Forces on an object in a flow.

given by the integral

$$D_F = \int_{A^*} (p_1 - p_2)\, dA \qquad \text{(Dab1)}$$

where A^* is the cross-sectional area of the object viewed from far upstream. Consequently the form drag coefficient based on the area A^*, C_{DF}, is

$$C_{DF} = \frac{1}{A^*} \int_{A^*} \frac{(p_1 - p_2)}{\frac{1}{2}\rho U^2}\, dA \qquad \text{(Dab2)}$$

and using the definition of the coefficient of pressure, C_p (see section (Bdg)), this can be written as

$$C_{DF} = \int_{A^*} (C_{p1} - C_{p2})\, \frac{dA}{A^*} \qquad \text{(Dab3)}$$

While we will return to more specifics in the section which follows, we note that since typical values of C_p on the surface of a bluff body vary from unity at the front stagnation point to smaller values away from that point and at the rear of the body, it follows from equation (Dab3) that typical values of C_{DF} for a bluff body in a high Reynolds number flow will be of order of 0.3 to 0.5.

In a similar way we can examine the skin friction drag, D_S, by dividing up the volume of the object into a series of cylinders of cross-sectional area, dA, perpendicular to the oncoming stream and the x axis as

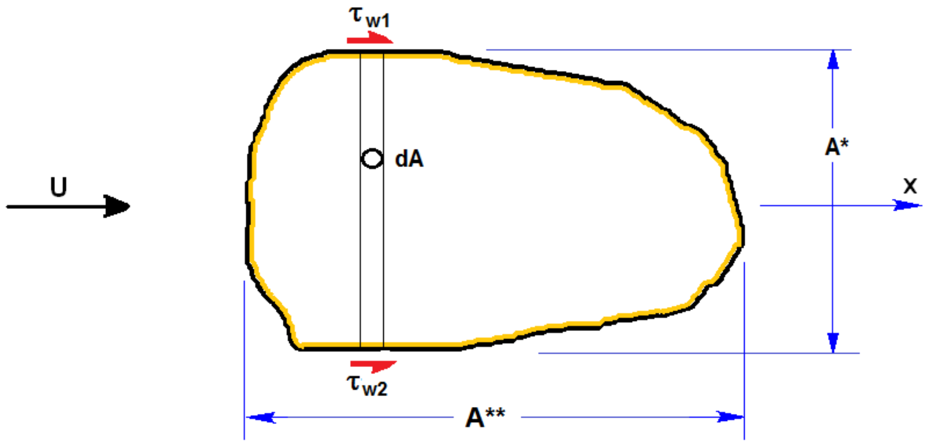

Figure 2: Forces on an object in a flow.

depicted in figure 2 and denoting the shear stresses acting on either end of the cylinder by τ_{w1} and τ_{w2}. It follows that the skin friction drag on the object, D_S, will be given by the integral

$$D_S = \int_{A^{**}} (\tau_{w1} + \tau_{w2}) \, dA \tag{Dab4}$$

where dA is the cross-sectional area of each of the elemental cylinders and A^{**} is the cross-sectional area of the object viewed from far away in the direction of the cylinder axes. Consequently the skin friction drag coefficient based on the area A^*, C_{DS}, is

$$C_{DS} = \frac{2}{A^*} \int_{A^{**}} \frac{(\tau_{w1} + \tau_{w2})}{\rho U^2} \, dA \tag{Dab5}$$

or

$$C_{DS} = \frac{2A^{**}}{A^*} \int_{A^{**}} \frac{(\tau_{w1} + \tau_{w2})}{\rho U^2} \, \frac{dA}{A^{**}} \tag{Dab6}$$

Note that the integrand contains the *skin friction coefficients* of the form $\tau_w/\rho U^2$ discussed in section (Bjf). In many practical high Reynolds number circumstances (such as in ship drag assessments), the skin friction drag is evaluated by using expressions for the *skin friction coefficients* taken from laminar and/or turbulent boundary layer analyses (see sections (Bjf) and (Bkk)) for which $\tau_w/\rho U^2$ is proportional to either $Re_s^{-\frac{1}{2}}$ or $Re_s^{-\frac{1}{5}}$ respectively with s being the surface distance from the front stagnation point. (Of course, adjustment factors of proportionality are used based on experimental measurement or flow computations.)

Whatever means are used to perform the integral in equation (Dab6), it can be seen that there is an additional factor that did not appear in the equivalent form drag equation, namely the factor $2A^{**}/A^*$, which effects the outcome. This factor is roughly proportional to the ratio of the length of the object, ℓ, to the breadth of the object, h. Consequently, even when the shear stresses are much smaller than the pressures, the skin friction drag may be comparable to the form drag when the object has an A^{**}/A^* value much greater than unity. Such can be the case for streamlined objects with $\ell/h \gg 1$.

The lift forces may, of course, be treated in a parallel manner as sketched in figure 3. For example, the form lift force, L_F, in a direction y perpendicular to U may be evaluated by dividing up the volume of the object into a series of cylinders of cross-sectional area, dA, with axes in the y direction as depicted in figure 3. Then, denoting the pressures acting on the ends of the cylinder by p_1 and p_2, it follows that the

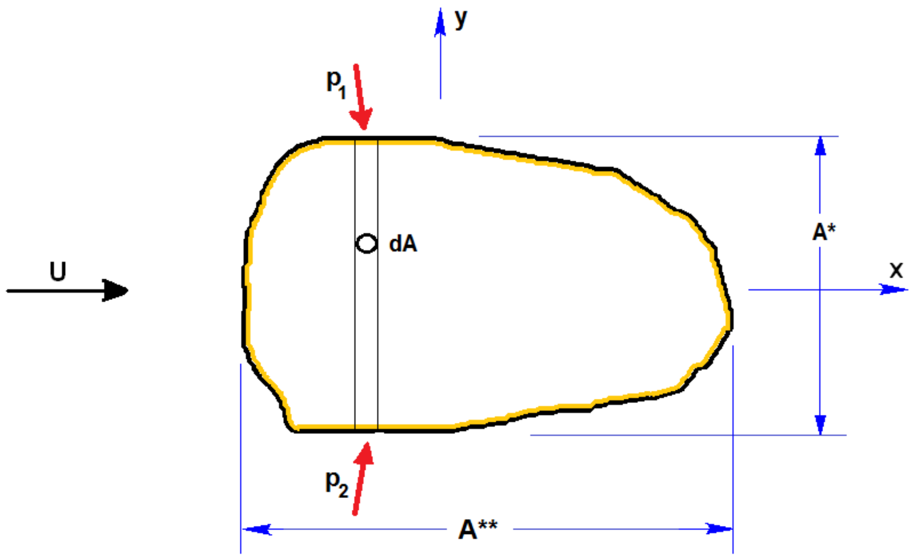

Figure 3: Forces on an object in a flow.

form lift on the object, L_F, in the y direction will be given by the integral

$$L_F \;=\; \int_{A^{**}} (p_2 - p_1)\; dA \tag{Dab7}$$

where, as before, A^{**} is the cross-sectional area of the object viewed from far away on the y direction. Consequently the form lift coefficient based on the area A^*, C_{LF}, is

$$C_{LF} \;=\; \frac{1}{A^*} \int_{A^{**}} \frac{(p_2 - p_1)}{\frac{1}{2}\rho U^2}\; dA \tag{Dab8}$$

and using the definition of the coefficient of pressure, C_p, this can be written as

$$C_{LF} \;=\; \frac{A^{**}}{A^*} \int_{A^{**}} (C_{p2} - C_{p1})\; \frac{dA}{A^{**}} \tag{Dab9}$$

Note that the appearance of the ratio, A^{**}/A^* in this expression suggests that the form lift can be much larger than the form drag for streamlined objects.

We could complete the set by detailing the skin friction lift but this is rarely of consequence.