Drag on a Sphere and Cylinder

It is useful to illustrate the complexity of the flow around an object, the changes with Reynolds number and the consequent changes in the drag by way of an example. The most studied example is the flow around a sphere or cylinder and hence we follow the developments of those flows as the Reynolds number increases from values very much less than unity to values much greater than unity. In some ranges we have to rely entirely on experimental observations; in others analytical solutions are available that allow detailed interpretation of the observations.

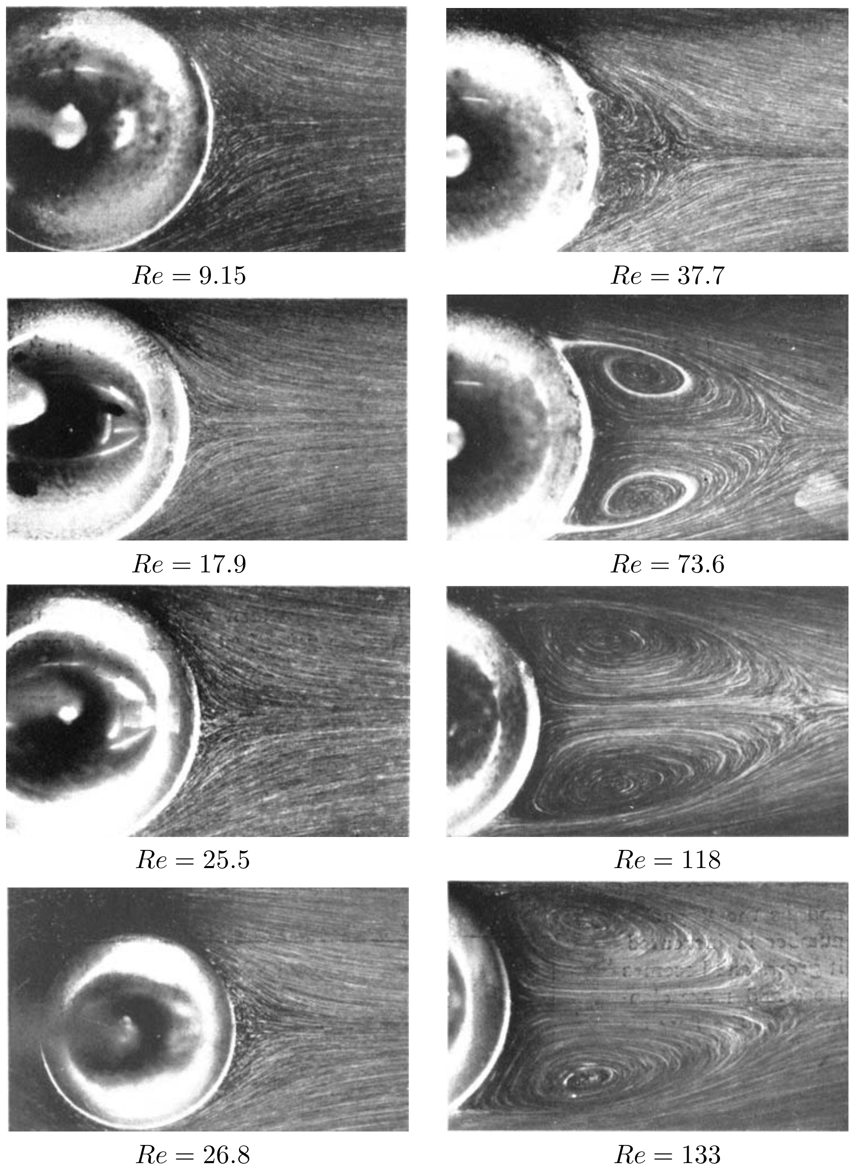

Figure 1: Streamlines of steady flow (from left to right) past a sphere at various Reynolds numbers (from Taneda 1956, reproduced by permission of the author).

The steady flow past a sphere at Reynolds numbers, $Re = Ud/\nu$, less than about 0.5 is accurately represented by the Stokes' flow solution detailed in section (Nec) and by the drag coefficient, $C_D = 24/Re$

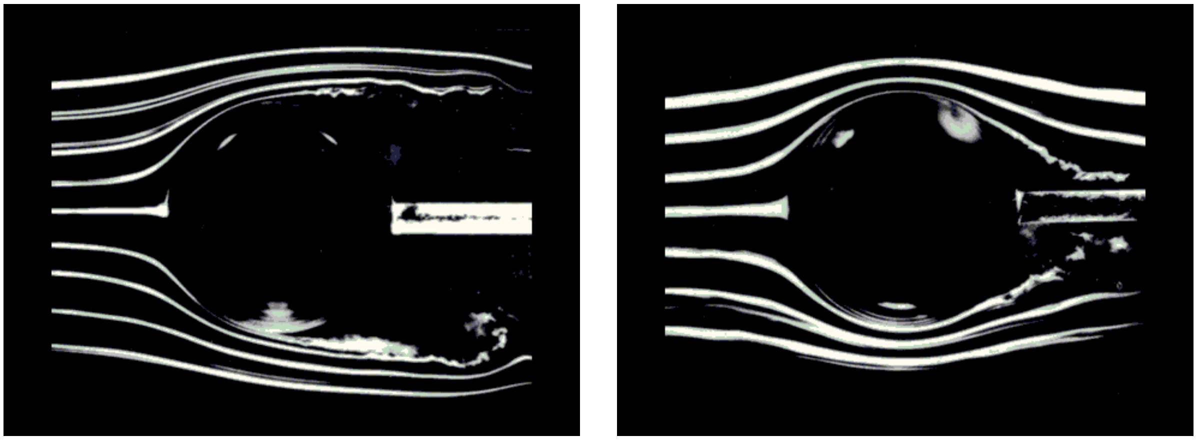

Figure 2: Smoke visualization of the nominally steady flows (from left to right) past a sphere showing, on the left, laminar separation at $Re = 2.8 \times 10^5$ and, on the right, turbulent separation at $Re = 3.9 \times 10^5$. Photographs by F.N.M.Brown, reproduced with the permission of the University of Notre Dame.

which that solution yields (the flow past a cylinder presents a special problem that is addressed elsewhere in sections (Blf) and (Blh).) Even for Reynolds numbers just above unity the experimentally observed streamlines in the flow behind a sphere are very similar to those of the Stokes' solution in that they are almost identical with those ahead of the sphere. However, as illustrated by the photographs of Figure 1, that symmetry begins to disappear at about $Re \approx 1$ and a vestigal "wake" begins to form near the rear stagnation point. Between $Re \approx 10$ and $Re \approx 50$ that wake grows into a recognizable "wake" consisting of a toroidal, recirculating eddy. Though, the flows past a sphere at Reynolds numbers between about 0.5 and several thousand have proven intractable to analytical methods (though numerical solutions are numerous) it should be noted that the approximate Oseen solution (see section (Blh)) does feature the formation of a wake-like structure.

With further increase in the Reynolds number above about $Re = 100$ this recirculating wake expands to the point where it approaches the width of the sphere. Defining locations on the surface by the angle from the front stagnation point, the separation point moves forward from about $130°$ at $Re = 100$ to about $115°$ at $Re = 300$. In the process the wake reaches a diameter comparable to that of the sphere when $Re \approx 130$. At this point the flow becomes unstable and the ring vortex that makes up the wake begins to oscillate (Taneda 1956). However, it continues to be attached to the sphere until about $Re = 500$ (Torobin and Gauvin 1959). At Reynolds numbers above about 500, vortices begin to be shed and be convected downstream. The frequency of vortex shedding has not been studied as extensively as in the case of a circular cylinder and seems to vary more with Reynolds number. In terms of the conventional Strouhal number, $St = 2fR/W$ (see sections (Df)), the vortex shedding frequencies, f, that Moller (1938) observed correspond to a range of St varying from 0.3 at $Re = 1000$ to about 1.8 at $Re = 5000$. Furthermore, as Re increases above 500 the flow develops a fairly steady "near-wake" behind which vortex shedding forms an unsteady and increasingly turbulent "far-wake." This process continues until, at a value of Re of the order of 1000, the flow around the sphere and in the near-wake again becomes quite steady. A recognizable boundary layer has developed on the front of the sphere and separation settles down to a position about $84°$ from the front stagnation point. Transition to turbulence occurs on the free shear layer, which defines the boundary of the near-wake and moves progressively forward as the Reynolds number increases. Then, quite abruptly at a Reynolds number of about $Re = 2 \times 10^5$, the turbulent shear layer reattaches to the body, resulting in a major change in the final position of separation ($\theta \cong 120°$) and in the form of the turbulent wake (Figure 2). This abrupt change is often referred to as the *drag crisis*. Associated with this change in flow pattern is a substantial change in the pressure distribution (see Figure 3 for the related pressure distribution change on a cylinder) and an associated decrease in the drag coefficient, C_D, from

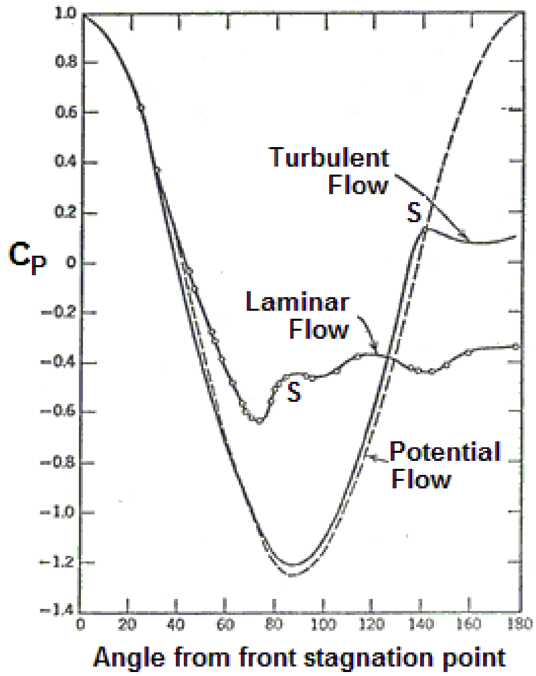

Figure 3: The pressure distribution on the surface of a cylinder for laminar and for turbulent boundary layer separation as well as the potential flow distribution. The coefficient of pressure, C_P, is plotted against the angle from the front stagnation point. The letter, S, denotes the separation location.

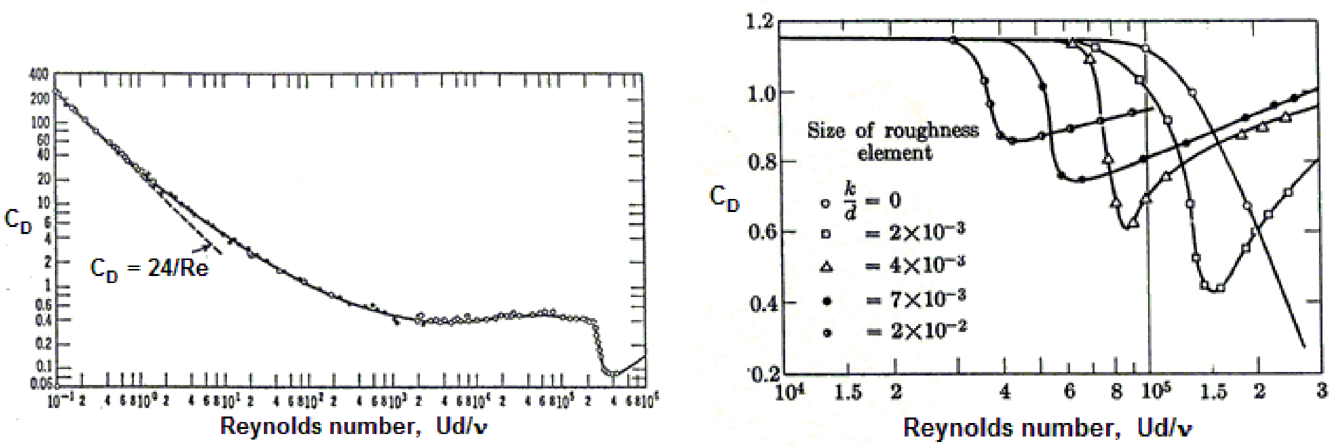

Figure 4: The drag coefficients measured for a sphere over a range of Reynolds numbers, Ud/ν, (left) and in the range of Reynolds numbers within which the separation point changes (right) for several roughnesses or surface finishes, k/d.

a value of about 0.5 in the laminar separation regime to a value of about 0.2 in the turbulent separation regime (Figure 4). That change is due to the change in the pressure distribution as exemplified by the measurements of the pressure distribution on the surface of a cylinder shown in Figure 3. The shift in the separation point causes an increase in the pressure at the separation point and consequently an increase in the pressure aft of the cylinder. A similar change occurs when drag crisis occurs with a sphere and this leads to the drop in the drag on a sphere at about $Re = 2 \times 10^5$ shown in Figure 4 (left). We also note that the conditions in the attached laminar boundary layer ahead of laminar separation can be significantly affected by the roughness of the surface of the object and therefore this roughness can affect the Reynolds number at which drag crisis occurs as demonstrated by the data of Figure 4 (right) in which the variation

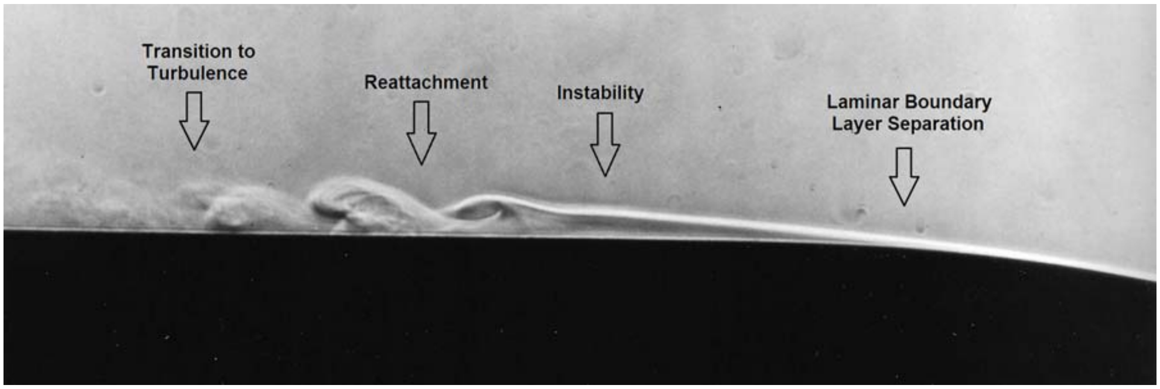

Figure 5: Shadowgraph photograph taken by V. Arakeri of the development of the boundary layer flow on the surface of an object in a water tunnel. The flow is from the right to the left.

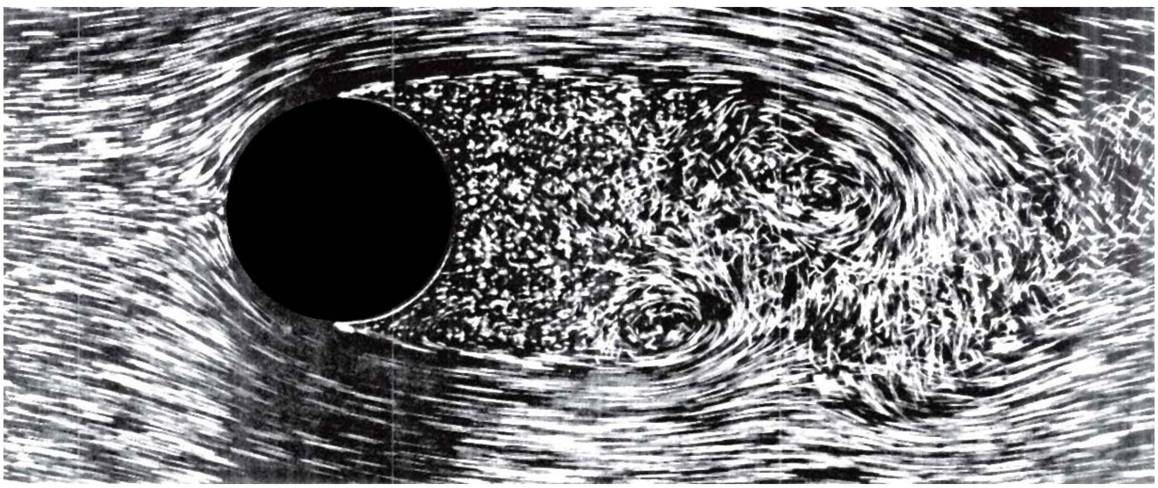

Figure 6: Flow around a cylinder at $Re = 2000$ visualized by air bubbles in the water. ONERA photograph by Werle and Gallon (1972).

in the drag crisis with surface roughness is evident (the obvious parameter which will pertain is the ratio of the surface roughness dimension to the diameter of the object, k/d).

To continue the tale of increasing Reynolds number above $Re \approx 2 \times 10^5$, it is now recognized that the transition to turbulent separation at $\theta \approx 120°$ is not as simple as described above. What appears to happen is that as the Reynolds number is increased within the range below $Re \approx 2 \times 10^5$ in which laminar separation occurs, transition to turbulence occurs in the separated shear layer (which is quite unstable). In the lower part of that range of Reynolds numbers that transition occurs at some distance downstream of the solid surface of the object. However, as the Reynolds number is increased the process of transition moves upstream. At the critical Reynolds number (about $Re \approx 2 \times 10^5$) the process of transition in the separated boundary layer encounters the solid surface and the resulting turbulent boundary layer "reattaches" as shown in Figure 5. It remains attached until turbulent separation occurs about $\theta \approx 120°$. What this means is that for Reynolds numbers greater than $Re \approx 2 \times 10^5$ the flow contains a hidden feature consisting of a laminar separation followed by turbulent transition and reattachment. This feature is known as a "laminar separation bubble" and is known to occur in the flow around the leading edge of some airfoils.

With further increase in the Reynolds number above $Re \approx 2 \times 10^5$ this laminar separation bubble shrinks in size. Some extremely high Reynolds number experiments with cylinders suggest that it may finally disappear at $Re \approx 10^8$ but this may well be a function of surface roughness.

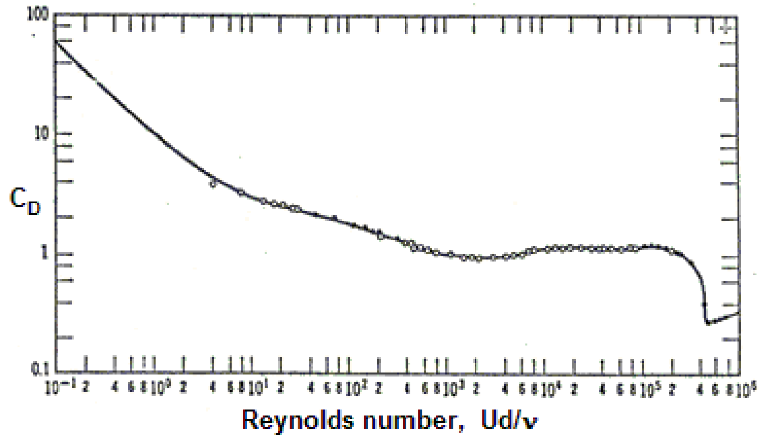

Figure 7: The drag coefficient, C_D, for a circular cylinder as a function of Reynolds number, Ud/ν.

Very similar developments occur in the flow around a sphere and a cylinder. For example Figure 6 shows the form of the flow around a cylinder at $Re = 2000$ and the formation and shedding of vortices in the wake.

Also Figure 7 shows the changes in the drag coefficient for a cylinder as the Reynolds number increases including the dramatic drop in the drag when the separation switches from laminar to turbulent.