Values of the Added Mass

Useful tabulations of the added mass for a variety of body shapes are presented by Patton (1965) and by Kennard (1967) and we will reproduce a sample of these results here. The vast majority of these results are theoretical and assume potential flow though it should be recalled that these results are also relevant to the relation between the forces and accelerations in a viscous flow at that first moment when the body and fluid are accelerating from rest. The reader will also note that all the quoted values refer to the diagonal components of the added mass matrix; there is virtually no data in the literature on the off-diagonal terms which seems like a serious oversight.

Theoretical values of the added mass for planar potential flow (added mass per unit length normal to the plane):

(A1) Circular cylinder of radius R:

$$M_{11} = M_{22} = \rho\pi R^2 \quad ; \quad M_{66} = 0 \tag{Bmbd1}$$

(A2) Elliptical cylinder with semi-axes a (in the 1 direction) and b (in the 2 direction):

$$M_{11} = \rho\pi b^2 \quad ; \quad M_{22} = \rho\pi a^2 \quad ; \quad M_{66} = \frac{1}{8}\rho\pi(a^2 - b^2)^2 \tag{Bmbd2}$$

(A2) Flat plate normal to 1 direction, width $2a$:

$$M_{11} = \rho\pi a^2 \quad ; \quad M_{22} = 0 \quad ; \quad M_{66} = \frac{1}{8}\rho\pi a^4 \tag{Bmbd3}$$

(A4) Circular cylinder of radius R sliding along a plane wall parallel to the 1 direction:

$$M_{11} = 2.29\rho\pi R^2 \tag{Bmbd4}$$

(A5) Rectangular cylinder of width a normal to the 1 direction and b normal to the 2 direction:

b/a	=	0.000	0.025	0.111	0.298	0.676	1.478	3.555	9.007	40.03
$M_{11}/\frac{1}{4}\rho\pi a^2$	=	1.00	1.05	1.16	1.29	1.42	1.65	2.00	2.50	3.50

(A6) Circular cylinder of radius a coaxial with a circular container of radius b:

$$M_{11} = M_{22} = \rho\pi a^2 \frac{(b^2 + a^2)}{(b^2 - a^2)} \tag{Bmbd5}$$

(A7) Circular cylinder of radius a in center of a square cylinder of side b when $a/b \ll 1$:

$$M_{11} = \rho\pi a^2 \left\{ 1 + 6.88 \left(\frac{a}{b}\right)^2 + \ ... \right\} \tag{Bmbd6}$$

(A8) Circular cylinder of radius a with center distance b from a plane wall when $a/b \ll 1$ (any direction):

$$M_{11} = \rho\pi a^2 \left\{ 1 + \frac{1}{2} \left(\frac{a}{b}\right)^2 + \ ... \right\} \tag{Bmbd7}$$

(A9) Circular cylinder of radius a midway between two plane walls parallel to the 1 direction and separated by b when $a/b \ll 1$:

$$M_{11} = \rho\pi a^2 \left\{ 1 + \frac{2\pi^2}{3}\left(\frac{a}{b}\right)^2 + \ ... \right\} \tag{Bmbd8}$$

Theoretical values of the added mass for three-dimensional potential flow:

(B1) Sphere of radius R:

$$M_{11} = M_{22} = M_{33} = \frac{2}{3}\rho\pi R^3 \quad ; \quad M_{44} = M_{55} = M_{66} = 0 \tag{Bmbd9}$$

(B2) Sphere of radius R with center distance a from a plane wall normal to the 1 direction when $R/a \ll 1$:

$$M_{11} = \frac{2}{3}\rho\pi R^3 \left\{ 1 + \frac{3}{8}\left(\frac{R}{a}\right)^3 + \ ... \right\} \tag{Bmbd10}$$

(B3) Sphere of radius R with center distance a from a plane wall parallel with the 1 direction when $R/a \ll 1$:

$$M_{11} = \frac{2}{3}\rho\pi R^3 \left\{ 1 + \frac{3}{16}\left(\frac{R}{a}\right)^3 + \ ... \right\} \tag{Bmbd11}$$

(B4) Sphere of radius R in the center of a spherical container of radius a:

$$M_{11} = \frac{2}{3}\rho\pi R^3 \left\{ \frac{a^3 + 2R^3}{a^3 - R^3} \right\} \tag{Bmbd12}$$

(B5) Prolate spheroid of major semi-axis a in the 1 direction and minor semi-axes b in the 2 and 3 directions:

a/b	=	1.00	1.50	2.00	2.51	2.99	3.99	6.01	8.01	9.97	∞
$M_{11}/\frac{4}{3}\rho\pi ab^2$	=	0.500	0.305	0.209	0.156	0.122	0.082	0.045	0.029	0.021	0.0
$M_{22}/\frac{4}{3}\rho\pi ab^2$	=	0.500	0.621	0.702	0.763	0.803	0.860	0.918	0.945	0.960	1.000
$M_{55}/\frac{4}{15}\rho\pi ab^2(a^2+b^2)$	=	0.000	0.094	0.240	0.367	0.465	0.608	0.764	0.840	0.883	1.000

and $M_{33} = M_{22}$, $M_{66} = M_{55}$ and $M_{44} = 0$.

(B6) Oblate spheroid of semi-axis a in the 1 direction and semi-axes b in the 2 and 3 directions ($b > a$):

b/a	=	1.00	1.50	2.00	2.50	3.00	4.00	6.00	8.00	10.00	∞
$M_{11}/\frac{4}{3}\rho\pi ab^2$	=	0.500	0.803	1.118	1.428	1.742	2.379	3.642	4.915	6.183	∞
$M_{22}/\frac{4}{3}\rho\pi ab^2$	=	0.500	0.384	0.310	0.260	0.223	0.174	0.121	0.092	0.075	0.000
$M_{55}/\frac{4}{15}\rho\pi ab^2(a^2+b^2)$	=	0.000	0.115	0.337	0.587	0.840	1.330	2.259	3.150	4.019	∞

and $M_{33} = M_{22}$, $M_{66} = M_{55}$ and $M_{44} = 0$.

(B7) Infinitely thin circular disc of radius R normal to the 1 direction:

$$M_{11} = \frac{8}{3}\rho R^3 \quad ; \quad M_{22} = M_{33} = M_{44} = 0 \quad ; \quad M_{55} = M_{66} = \frac{16}{45}\rho\pi R^5 \tag{Bmbd13}$$

(B8) Infinitely thin elliptical disc of major semi-axis a and minor semi-axis b with disc normal to the 1 direction:

a/b	1.00	1.50	2.00	3.00	4.00	6.00	8.19	10.43	14.30	∞
$M_{11}/\frac{4}{3}\rho\pi ab^2$	0.637	0.748	0.826	0.900	0.933	0.964	0.978	0.985	0.991	1.000

(B9) Infinitely thin rectangular plate of major sides a and b ($a > b$) with plate normal to the 1 direction:

a/b	1.00	1.50	2.00	3.00	4.00	∞
$M_{11}/\frac{1}{4}\rho\pi ab^2$	0.478	0.680	0.840	1.000	1.000	1.000

Added masses from experiments on accelerating bodies and/or flows:

Experiments on accelerating bodies immersed in a fluid and/or accelerating flows past bodies (see, for example, Keulegan and Carpenter 1958, Skop *et al.* 1976, or Sarkaya 1977) must necessarily try to separate the inertial forces from the drag forces on the body (see also Wehausen 1971). There is no fundamentally rigorous way to do this for all Reynolds numbers and, as we shall see, the problem becomes particularly complicated at the intermediate Reynolds numbers at which most of the experiments have been performed. Mostly, researchers have resorted to the heuristic approach of arbitrarily adding fluid inertial forces to the fluid drag forces. This is usually done in the form of what is known as Morison's equation (see Morison *et al.* 1950) in which the total force, F_i^T, on a body in a fluid otherwise at rest is expressed as

$$F_i^T = -M_{ij}\frac{dU_j}{dt} - \frac{1}{2}\rho A C_{ij}\left\{|U_j|\right\}^2 \qquad \text{(Bmbd14)}$$

where U_j is the velocity of the body, C_{ij} is a lift and drag coefficient matrix and A is the typical cross-sectional area of the body on which those coefficients are based. This equation is normally quoted for just one direction and written as

$$F^T = -M\frac{dU}{dt} - \frac{1}{2}\rho A C_D|U|^2 \qquad \text{(Bmbd15)}$$

where M is the added mass and C_D, U, and dU/dt refer to a drag coefficient, the velocity and acceleration in line with the force. This empirical construction might be more useful if the best values for M and C_D varied little from one specific motion to another. However, Keulegan and Carpenter (1958), for example, observed that this was not the case and that substantial changes in the effective values of M and C_D in their fixed cylinder experiments occurred as the period, T, of their sinusoidal applied fluid motion was increased. They sought to find correlation with the parameter, $U_M T/2R$, where U_M is the magnitude of the free stream velocity and R is the radius of the cylindrical body. They found that the effective value of the added mass, M, was close to the potential flow value of $\rho\pi R^2$ per unit length for $U_M T/2R$ below about 5 but decreased rapidly with increasing $U_M T/2R$ becoming negative for a range of $U_M T/2R$ between about 10 and 20 (we have subtracted the displaced fluid mass from their results in order to obtain the true added mass in line with the discussion in the section on fluid acceleration). With still further increase in $U_M T/2R$ positive values for the effective added mass similar to those for $U_M T/2R < 5$ are recovered. The corresponding variations in the effective drag coefficient exhibit a large increase for the intermediate range of $10 < U_M T/2R < 20$. No systematic variations with Reynolds number could be discerned though the range of Reynolds numbers, $2U_M R/\nu$ (ν being the kinematic viscosity), of their experiments, namely 5000 to 30,000 is one in which substantial unsteadiness is observed even in nominally steady flows. Experiments with flat plates exhibited a different pathological behavior.

Skop, Ramberg and Ferer (1976) also carried out sinusoidally oscillating flow experiments except that the body rather than the fluid flow was oscillated. Their results disagree substantially with those of Keulegan and Carpenter despite the fact that they cover roughly the same range of Reynolds numbers. For

$1 < U_M T/2R < 12$ they found that the fluid inertial force agreed well with the potential flow value. They also found that the effective drag coefficient could be accurately predicted by considering the instantaneous Reynolds number at each point in the cycle, using some appropriate form for the corresponding steady flow drag coefficient and synthesizing the overall effective drag coefficient for the whole cycle.

Sarpkaya (1977) carried out experiments in which he oscillated a cylinder in the lateral direction in a uniform stream of velocity, U. He observed pathological behavior when $UT/2R$ was between about 3 and 10 which is not unexpected since this range corresponds to the vortex shedding frequency in the nominally steady flow. Moreover, in one of the few observations of off-diagonal effects he found that the lateral motion caused oscillations in the drag amounting to about 7% of the time-averaged drag.