Joukowski Airfoils

In section (Bmbb), the following expression for the added mass matrix in potential flow was derived

$$M_{jk} \;=\; -\rho \int_S \phi_j \frac{\partial \phi_k}{\partial n} dS \tag{Bmbe1}$$

where S is the surface of the body, n is the outward normal to that surface and ϕ_j and ϕ_k are the velocity potentials on the surface of the object whose added mass matrix we seek; specifically they are the velocity potentials of the steady potential flows due to translation of the foil with unit velocity in the j and k directions (and zero fluid velocity far from the foil). Moreover, the condition of zero normal velocity on the surface means that

$$\frac{\partial \phi_k}{\partial n} \;=\; n_k \tag{Bmbe2}$$

where n_k is the component of the unit outward normal in the k direction. Substituting this into equation (Bmbe1) it follows that

$$M_{jk} \;=\; -\rho \int_S \phi_j n_k dS \tag{Bmbe3}$$

Now the steady potential flow past a Joukowski airfoil was detailed in section (Bged) where the velocity potential on the surface of the foil, ϕ^{**}, was given by equation (Bged19) as

$$\frac{\phi^{**}}{U} \;=\; 2R\left[\cos\left(\theta-\alpha\right) - \left(\theta-\alpha\right)\sin\left(\alpha+\beta\right)\right] \tag{Bmbe4}$$

where the surface begins at the trailing edge at $\theta = -\beta$ and ends at the trailing edge at $\theta = 2\pi - \beta$. However, ϕ^{**}, is not the surface velocity potential that is needed for present purposes. As described in section (Bged), it is the velocity potential for a flow in which the foil is fixed in position and the flow far from the foil is a uniform stream of velocity U inclined at an angle α to the ξ direction. The velocity potential we need for present purposes is that in which the flow far from the foil is at rest and the foil is moving with a velocity U at an angle α to the ξ direction. The adjustment is simply a matter of reversing the sign of the right hand side of equation (Bmbe4) and adding a uniform stream component of magnitude, U, and inclination α to obtain the following adjusted velocity potential, ϕ^{***}:

$$\frac{\phi^{***}}{U} \;=\; \xi e^{-i\alpha} - 2R\left[\cos\left(\theta-\alpha\right) - \left(\theta-\alpha\right)\sin\left(\alpha+\beta\right)\right] \tag{Bmbe5}$$

It follows from this that the earlier defined ϕ_j in equation (Bmbe3) is given by

$$\left[\phi_j\right]_{j=1} \;=\; \left(\frac{\phi^{***}}{U}\right)_{\alpha=0} \qquad \text{and} \qquad \left[\phi_j\right]_{j=2} \;=\; \left(\frac{\phi^{***}}{U}\right)_{\alpha=\pi/2} \tag{Bmbe6}$$

where the $j=1$ and $j=2$ directions correspond to the directions of the ξ and η axes.

In order to perform the surface integral in equation (Bmbe3) we note that the relation between a surface element, ds, in the ζ plane and a surface element, $d\theta$, in the z plane is

$$(ds)^2 \;=\; (d\xi)^2 + (d\eta)^2 \qquad \text{so that} \qquad \frac{ds}{d\theta} \;=\; R\left|1-\frac{a^2}{z^2}\right| \tag{Bmbe7}$$

Moreover, since the components of the unit normal to the surface, n_k, are given by

$$n_k = \left[\frac{(1 - a^2/z^2)e^{i\theta}}{|1 - a^2/z^2|} \right]_k \quad \text{(Bmbe8)}$$

it follows that

$$n_k \frac{ds}{d\theta} = \left[R \left(1 - \frac{a^2}{z^2} \right) e^{i\theta} \right]_k \quad \text{(Bmbe9)}$$

Thus, finally, the integrand in equation (Bmbe3) is completely defined and integrations can be performed numerically to determine the added masses, M_{jk}.

We define non-dimensional added mass coefficents, M^{**}_{jk}, as

$$M^{**}_{jk} = \frac{M_{jk}}{\rho s c^2} \quad \text{(Bmbe1)}$$

where c and s are the chord and span of the foil. Then the diagonal added mass coeffient, M^{**}_{11}, in the direction of the chord line is shown in Figure 1, and the diagonal added mass coefficient, M^{**}_{22}, in the direction normal to the chord line is plotted in Figure 2. These coefficients are plotted for various foil

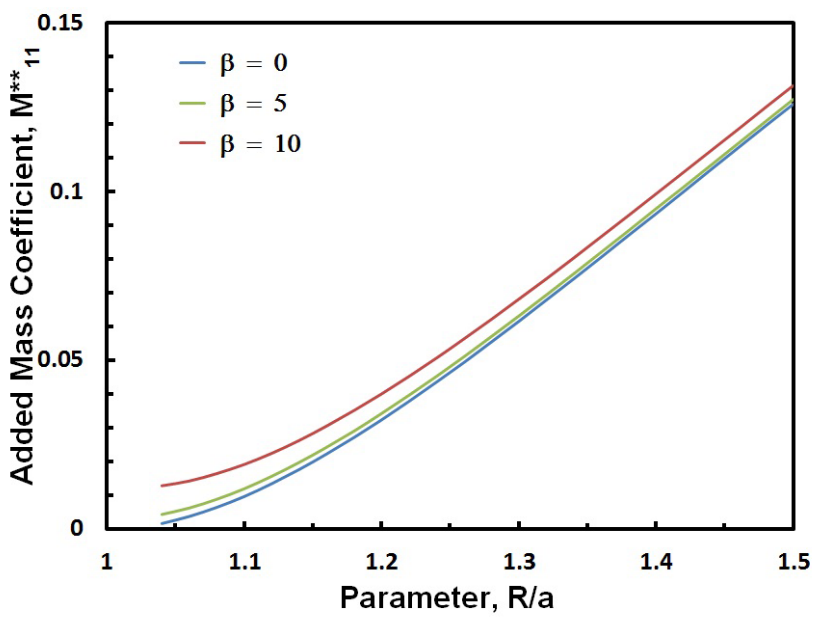

Figure 1: The added mass, M^{**}_{11}, for a range of different Joukowski airfoils.

thickness parameters, R/a, and for three different angles β (see section (Bged) for the corresponding foil geometries and lift coefficients in steady flow).

Notice that when $R/a \rightarrow 1$ and $\beta = 0$ the foil becomes a flat plate for which $M^{**}_{11} \rightarrow 0$ and $M^{**}_{22} \rightarrow \pi/4 = 0.785$ in accord with the results listed in section (Bmbd).

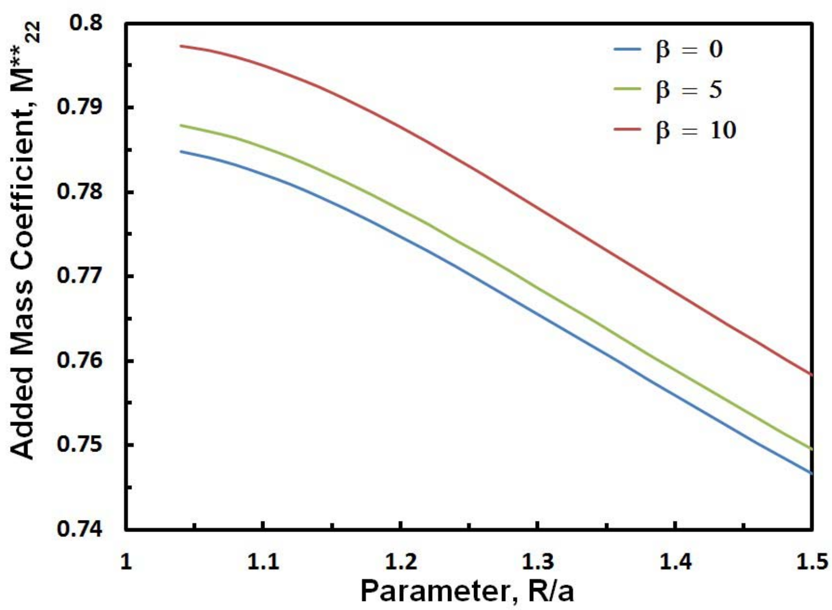

Figure 2: The added mass, M_{22}^{**}, for a range of different Joukowski airfoils.