

## Turbulence Development

The flow instabilities that are the focus of this introduction to turbulence cause very small disturbances or imperfections in the flow to be amplified. Nominally steady flow in this context is one in which the time-averaged pressure and velocity is not changing with time,  $t$ , though it may be a function of position,  $x_i$ , within the flow. In such flows we will seek to evaluate the rate of growth of the disturbances with position, the rate of growth as they are convected downstream. However, we shall also identify the rate of growth of the disturbances with time in non-steady flows.

In most instances it is difficult to identify the precise source of the very small imperfections or “noise” that initiates the process. However, there seem to be at least three different possible sources of the noise:

- Imperfections in the incoming fluid stream known as “free stream” turbulence. It is essentially impossible to eliminate the free-stream turbulence in a water tunnel or wind tunnel in which experiments are conducted.
- Surface roughness of the object or objects that create the flow; again it is essentially impossible to eliminate surface roughness effects.
- Acoustic noise and structural vibration in the test facility.

All of this “noise” will consist of a range of frequencies,  $f$ , and both the spectra and amplitude,  $A$ , of this noise is important in an attempt to construct how it will be amplified by the flow.

As we shall see, it transpires that many of the flows with which we will be concerned, are highly selective amplifiers. That is to say the amplification rate,  $dA/dx$ , is a strong function of the frequency,  $f$  ( $x$  could be the longitudinal coordinate in a pipe or  $s$  in a boundary layer flow). At least during the early phase of amplification, only a narrow range of frequency is amplified as depicted in Figure 1. Later,

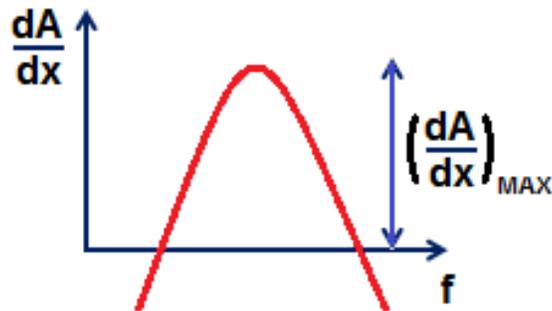


Figure 1: Representation of the selective amplification of noise in a pipe or boundary layer.

as the amplitude increases to a level where frequency dispersion becomes significant, the waves “break” to generate a spectrum of frequencies. This progress in the amplification process can be observed in measurements of the instantaneous velocities in a flow undergoing transition to turbulence. As we shall see later, the rate of amplification is also a function of the Reynolds number and the shape of the velocity profile.

The early selective amplification of the flow perturbations is apparent in measurements of the instantaneous velocities in laminar boundary layers as exemplified by the measurements reproduced in Figure 2. It can be

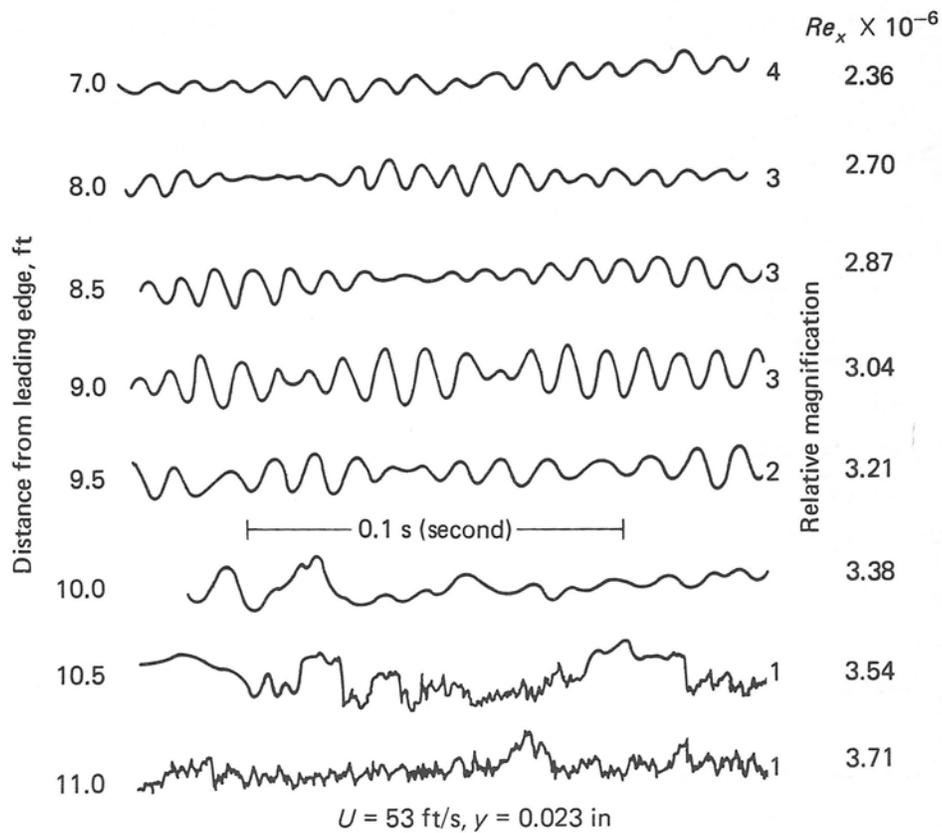


Figure 2: Unsteady velocity fluctuations with time at various longitudinal locations (distances from the leading edge on the left) in a transitioning boundary layer shown at various amplitude magnifications (shown on the right). The corresponding Reynolds numbers,  $Re_x$ , are also shown on the right. After Schubauer and Skramstad (1947).

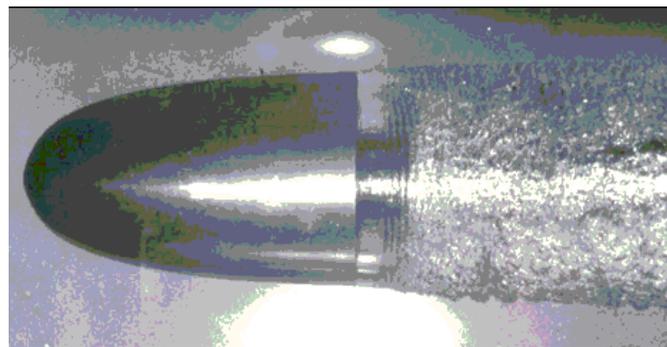


Figure 3: Flow of water past an ogival headform showing the growing Tollmein-Schlichting waves on the interfacial layer just after separation (Brennen (1970)).

seen that the velocity measurements just a short distance downstream of the location where the boundary layer first becomes unstable exhibit a regular sinusoidal variation with time. However, as the disturbance progresses downstream note that the waves begin to break-up and a range of frequency content becomes apparent. Eventually this frequency dispersion results in an approach to fully-developed turbulence with frequency content described later in section (Bkf).

The wave perturbations that occur early in the transition process are often called Tollmein-Schlichting waves, after of two of the early researchers in the area. These waves can also be seen visually in the photographs presented in Figures 3 to 6. These photographs show the flow of water past various headforms in a water



Figure 4: Flow of water past a spherical headform showing the growing Tollmein-Schlichting waves on the interfacial layer just after separation (Brennen (1970)).

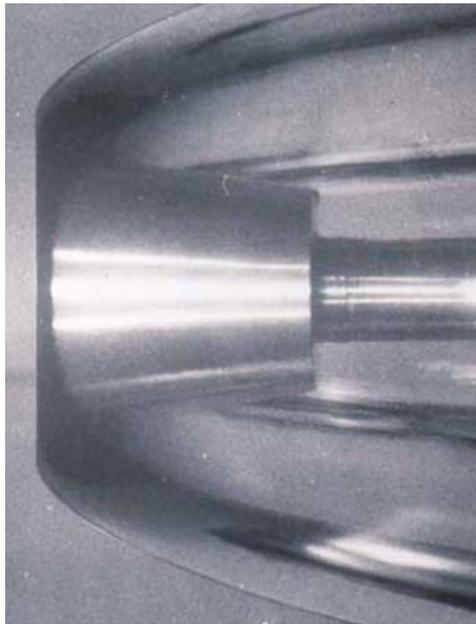


Figure 5: Flow of water past a disc headform showing no Tollmein-Schlichting waves or transition of the interfacial layer (Brennen (1970)).

tunnel. The tunnel velocity is from right to left and the wake behind the headforms is filled with air. Therefore the boundary layer that develops where the flow is in contact with the headform becomes a layer with a free surface following the point of separation of the flow from the headform. The new velocity profile following separation is much more unstable than the attached boundary layer (because it now has a point of inflection) and so becomes quite unstable. In Figures 3 and 4 you can see the growing Tollmein-Schlichting waves on the free surface just after separation. The waves grow and transition to a turbulent interfacial layer for the remainder of the flow downstream.

Two additional photographs from the same series shown some other features of the transition process. Figure 5 involves a disc headform; because of the greater fluid acceleration on the wetted surface, the

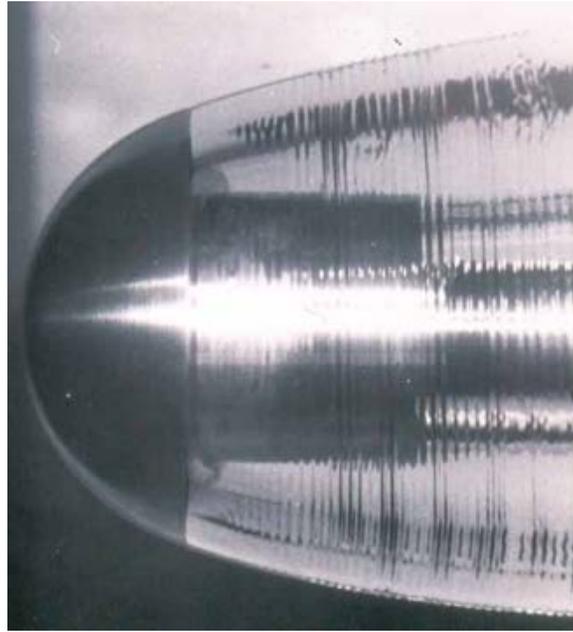


Figure 6: Flow of water past a spherical headform showing Tollmein-Schlichting waves whose growth is very limited (Brennen (1970)).

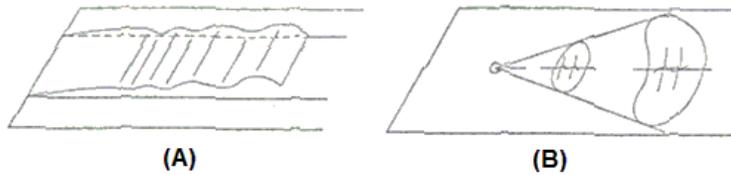


Figure 7: Two forms of growing turbulence disturbances.

separating boundary layer is much thinner in this case and no waves or transition occur. Figure 6 shows another case for the spherical headform but this at a smaller velocity than Figure 4.

It is also important to note that the three-dimensionality of the Tollmein-Schlichting waves can take several forms. In the above photographs the waves are primarily two-dimensional as depicted by type (A) in Figure 7. However, in other cases, as in an attached boundary layer, they appear to grow as turbulent spots as depicted by type (B) in Figure 7 and there may be other basic geometric configurations. The type that occurs appears to depend on the form of the velocity profile.

In an attached turbulent boundary layer, these spots manifest themselves as intermittent “bursts” of turbulence as can be seen in the flow-visualization photograph of Figure 8. These bursts can be recognized in velocity measurements as sketched on the right in Figure 9 and quantified in the graph on the left by an intermittency parameter.

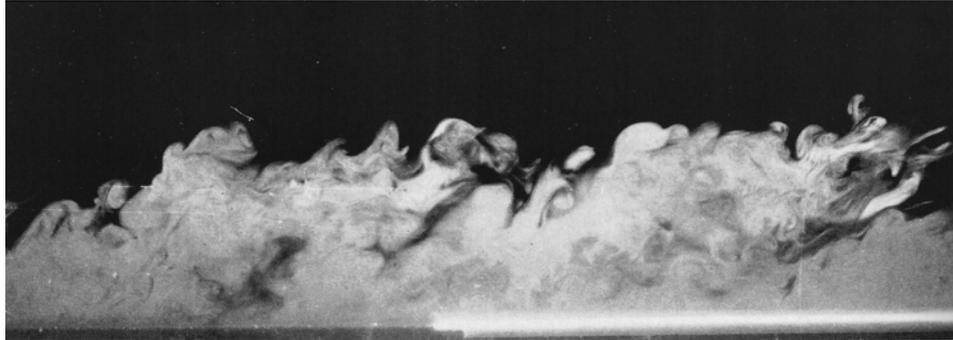


Figure 8: Visualization of a turbulent boundary layer on a wall at a point where the Reynolds number based on the momentum thickness is about 4000. After Falco (1977).

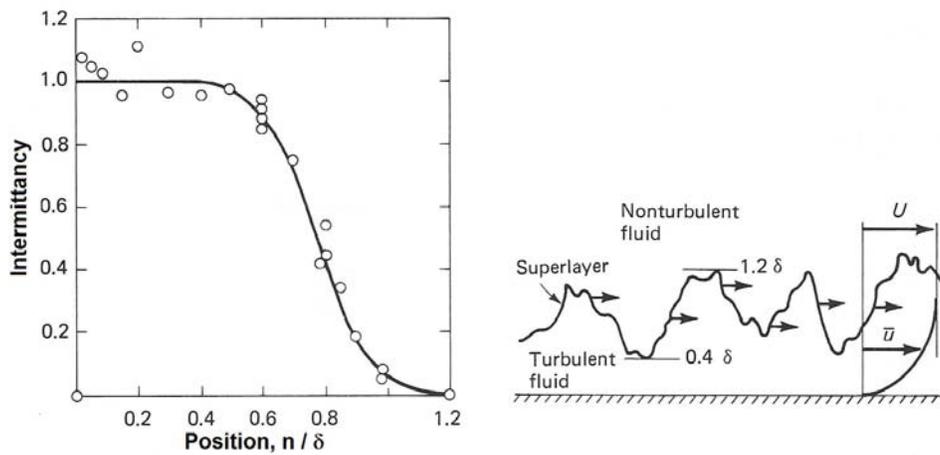


Figure 9: Intermittency in a turbulent boundary layer as sketched on the right and quantified in the graph on the right by the intermittency parameter. After Klebanoff (1955).