Amplification of Disturbances

Given the information on amplification rates provided by the stability analyses of section (Bkd) it is possible to envisage computing the amplitude of the Tollmein-Schlicting waves as the flow proceeds downstream and hence, perhaps, being able to estimate where that amplitude becomes comparable to the wavelength so that the waves begin to break up and generate turbulence. Such calculations have, indeed, been formulated in an effort to predict transition to turbulence. For example, if in a spatial amplification case, we denote the disturbance amplitude for a particular disturbance frequency, ω_R, by a then analyses such as those pursued in sections (Bkc) and (Bkd) would provide the disturbance amplification rate, $-k_I$ in the x direction where, in the present discussion, k_I is negative. If would follow that

$$\frac{1}{a}\frac{da}{dx} \;=\; -k_I(x) \tag{Bke1}$$

and therefore by integration

$$a(x) \;=\; a(x_0)\; exp\left\{\int_{x_0}^{x} -k_I(x)\; dx\right\} \tag{Bke2}$$

Then, if one knew the amplitude of the initial disturbance, $a(x_0)$, when that frequency first became unstable (and the location, x_0, where it first became unstable) one could evaluate the amplitude of that frequency at all locations downstream. However, there are three serious difficulties with such a calculation. The first is the difficulty of determining the amplitude where the flow first becomes unstable for usually this is a very small quantity. The second is the difficulty of knowing what amplitude constitutes the initiation of turbulence. And the third is the fact that the analyses of sections (Bkc) and (Bkd) are linear analyses and non-linear effects will undoubtedly become important as the end of this growth process is approached.

Despite these difficulties, methods of the above type have been developed in an effort to predict transition to turbulence, for example by A.M.O.Smith and his colleagues at Douglas Aircraft. Smith recommended a rule of thumb that transition would occur when $a(x)/a(x_0) = e^n$ where n is of the order of 8 or 9. This can only be regarded as a very crude estimate but one that was based on experimental observations.

Though quantifying the progress toward turbulence is difficult, more is known of the qualitative mechanisms that occur during the process of transition to turbulence. Though these mechanisms can be quite flow specific, the general pattern is that some flow instability (such as the instability analyzed in sections (Bkc) and (Bkd)) leads to some fairly large scale disturbance(s) or “eddies” in the flow field. As these disturbances gather energy from the mean flow, they begin to spawn smaller disturbances or eddies which, in turn spawn even smaller eddies. This process ends because, eventually, the eddies reach a size for which viscous effects become critical and the very small eddies are damped out by viscosity. Eventually, the spectrum of spatial or temporal eddy sizes reaches a “fully-developed” state in which energy is continually cascading down to smaller-sized eddies and the disturbance energy for any one size of eddy becomes relatively constant. We discuss more of the details of this process in the next section.

The large scale disturbances that occur at the beginning of the above process of transition can take a number of forms. One are the Tollmein-Schlicting waves of sections (Bkc) and (Bkd). Another common one are the eddies shed due an instability in the flow around a bluff body.