

Hydraulic Jumps

In the preceding section we demonstrated how a compressive wave in open channel flow would evolve into a wave of permanent form known variously as a *hydraulic jump* or *bore*. To analyze the details of this phenomena it is most useful to do so in a frame of reference fixed in the jump as shown in Figure 1. In this frame of reference, the depth-averaged velocities upstream and downstream of the jump are denoted by u_1 and u_2 respectively and the upstream and downstream depths are denoted by H_1 and H_2 . Then we apply the equations of conservation of mass and the linear momentum theorem to a control volume consisting of the upstream and downstream boundaries shown in Figure 1 that extend far above the liquid and the solid boundary at the base of the flow. Then, assuming for simplicity that the breadth of the flow normal

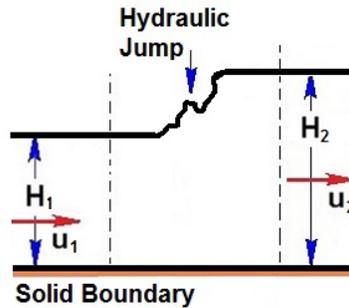


Figure 1: Flow in a frame fixed in the hydraulic jump.

to the sketch is unity, conservation of mass for this flow which is steady in the frame of reference chosen requires that

$$u_1 H_1 = u_2 H_2 = Q \quad (\text{Bpc1})$$

where Q is used to denote the volume flow rate (per unit breadth). To apply the linear momentum theorem in the direction of flow we note that the hydrostatic force on the upstream boundary is $\rho g H_1^2/2$ (where g is the acceleration due to gravity and ρ is the liquid density assumed constant) while that on the left side is $\rho g H_2^2/2$ so that the net hydrostatic force is $\rho g (H_1^2 - H_2^2)/2$ in the direction of flow. We neglect any viscous forces that may act at the solid boundary (see section (Bpe)). The momentum flux in through the left hand boundary is $\rho u_1^2 H_1$ and out through the right-hand boundary is $\rho u_2^2 H_2$. Consequently the linear momentum theorem yields

$$\rho u_2^2 H_2 - \rho u_1^2 H_1 = \frac{1}{2} \rho g (H_1^2 - H_2^2) \quad (\text{Bpc2})$$

which can be written as

$$\frac{Q^2}{H_1 H_2} (H_1 - H_2) = \frac{g}{2} (H_1 - H_2) (H_1 + H_2) \quad (\text{Bpc3})$$

Therefore *either* $H_1 = H_2$ and there is no jump at all *or*

$$\frac{2Q^2}{g H_1 H_2} = (H_1 + H_2) \quad (\text{Bpc4})$$

which is a quadratic in H_2 and whose solution is

$$H_2 = -\frac{H_1}{2} \pm \left[\frac{H_1^2}{4} + \frac{8Q^2}{g H_1} \right]^{\frac{1}{2}} \quad (\text{Bpc5})$$

Clearly the only realistic choice is the positive sign and so the downstream depth must be given by

$$H_2 = -\frac{H_1}{2} \left[-1 + \left\{ 1 + \frac{8Q^2}{gH_1^3} \right\}^{\frac{1}{2}} \right] \quad (\text{Bpc6})$$

This relation between the upstream and downstream depths (or between the upstream and downstream velocities since $H_2/H_1 = u_1/u_2$) can be written in non-dimensional terms using $Fr_1 = u_1/(gH_1)^{1/2}$ and $Fr_2 = u_2/(gH_2)^{1/2}$:

$$Fr_2 = Fr_1 \left[\frac{-1 + (1 + 8Fr_1^2)^{\frac{1}{2}}}{2} \right]^{-\frac{3}{2}} \quad (\text{Bpc7})$$

The Froude number ratio, Fr_2/Fr_1 and the depth ratio, H_1/H_2 , are both plotted in Figure 2 as functions of the upstream Froude number, Fr_1 . It is evident from the above relations and from the data that

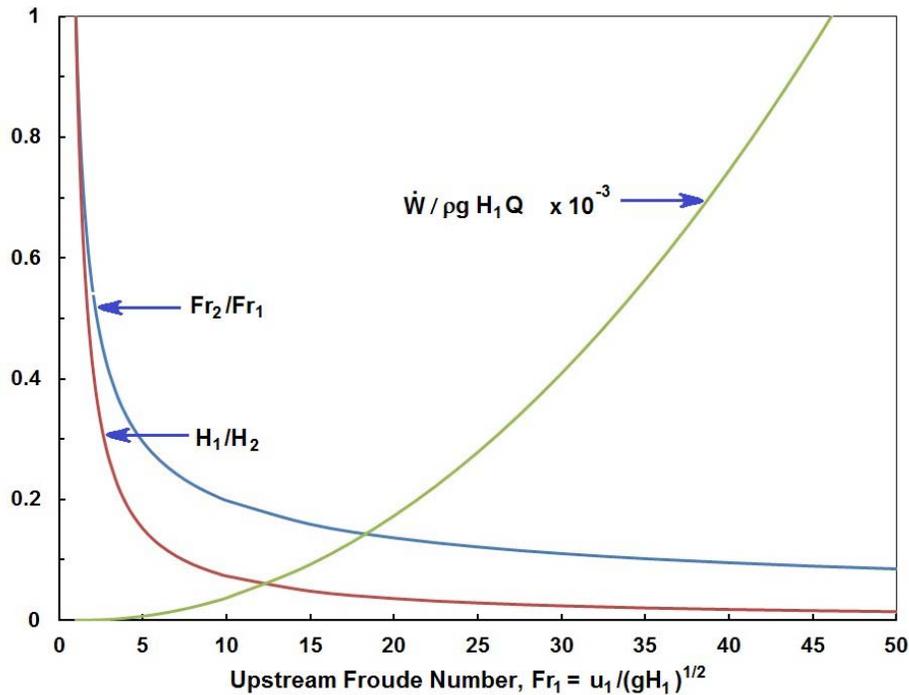


Figure 2: The ratio of the Froude numbers, Fr_2/Fr_1 and the depths, H_1/H_2 , across a hydraulic jump as a function of the upstream Froude number, Fr_1 . Also shown is the loss of energy per unit breadth, \dot{W} , plotted as $\dot{W} / \rho g H_1 Q \times 10^{-3}$.

- The upstream Froude number is always greater than unity.
- The downstream Froude number is always less than one, so that just like a shock wave in a compressible fluid in which the upstream flow relative to the shock is always supersonic and the downstream flow relative to the shock is always subsonic, in a hydraulic jump the upstream flow relative to the jump is always supercritical whereas the downstream flow relative to the jump is always subcritical.
- There is always a dissipation of kinetic energy within the hydraulic jump just as there is a dissipation of kinetic energy in a compressible fluid shock wave. This dissipation of energy occurs through the viscous dissipation and turbulence that always takes place within a hydraulic jump.

To evaluate the energy losses in the flow through a hydraulic jump consider a streamtube through the jump as exemplified by that shown in Figure 3. If we denote the atmospheric pressure above the free surface by

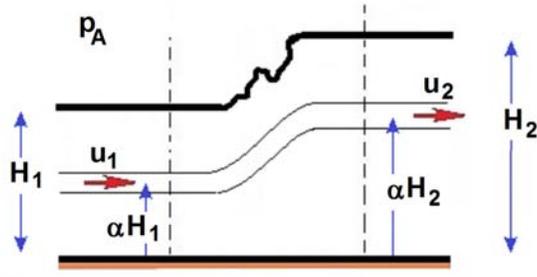


Figure 3: Streamtube through a hydraulic jump.

p_A , then the static pressure, p_1 , in the tube that enters the jump at an elevation of αH_1 ($0 < \alpha < 1$) will be $p_A + (1 - \alpha)\rho g H_1$ and therefore the total pressure, p_1^T , will be

$$p_1^T = p_A + (1 - \alpha)\rho g H_1 + \alpha\rho g H_1 + \frac{1}{2}\rho u_1^2 = p_A + \rho g H_1 + \frac{1}{2}\rho u_1^2 \quad (\text{Bpc8})$$

using the solid boundary as the elevation reference level. Similarly the total pressure in the streamtube downstream of the jump will be

$$p_2^T = p_A + \rho g H_2 + \frac{1}{2}\rho u_2^2 \quad (\text{Bpc9})$$

so the total pressure drop in the stream tube as it passes through the hydraulic jump will be

$$p_1^T - p_2^T = \rho g(H_1 - H_2) + \frac{1}{2}\rho(u_1^2 - u_2^2) \quad (\text{Bpc10})$$

and is the same for any streamtube since it is independent of the chosen location, α . Using $Q = u_1 H_1 = u_2 H_2$ and the expression (Bpc4) for Q this may be written as

$$p_1^T - p_2^T = \frac{\rho g(H_2 - H_1)^3}{4H_2 H_1} \quad (\text{Bpc11})$$

and therefore the energy loss in the hydraulic jump per unit breadth, $\dot{W} = (p_1^T - p_2^T)Q$, is given by

$$\frac{\dot{W}}{\rho g H_1 Q} = \frac{(H_2 - H_1)^3}{4H_2 H_1^2} \quad (\text{Bpc12})$$

This dimensionless energy loss is also plotted in Figure 3 and demonstrates the dramatic rise in the energy loss as the magnitude of the jump increases. In most practical circumstances this energy loss manifests itself as massive turbulence and mixing within the jump though very small scale jumps can be quite smooth as the energy is dissipated by viscous effects within the jump.

Typically hydraulic jumps in rivers or weirs occur when the flow proceeds over a restriction (see section (Bpd)), becomes supercritical and accelerates downward, ending in a hydraulic jump through which it transitions back to subcritical. The photograph in Figure 4 (left) is a typical example of this commonly observed phenomenon. Interestingly jumps can also occur in other, non-liquid media; Figure 4 (right) is an example of a hydraulic jump in the flow of granular material down an inclined chute (Brennen *et al.* (1983); the supercritical layer of flow upstream of the jump is thin and quite fast so that it is not well captured in the photograph).

Another common appearance of a hydraulic jump is the *tidal bore* that is formed in a narrowing estuary as an incoming tide is focussed by the decreasing cross-sectional area of the estuary. Examples occur around the world, with some estuaries being notorious because of their particular topography. Examples are the bores that are formed in Morecambe Bay (Figure 5, left) on the west coast of England and in the Bay of Fundy (Figure 5, right) in eastern Canada.

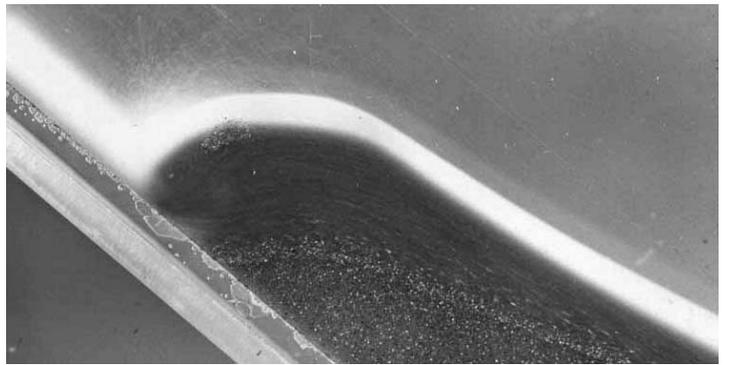


Figure 4: Left: Hydraulic jump in a river. Right: Hydraulic jump in granular material flow (from Brennen, Sieck and Paslaski, 1983).



Figure 5: Left: Tidal bore in Morecambe Bay, UK (photograph by Arnold Price). Right: Tidal bore in Moncton, Bay of Fundy, Canada (photograph by Charles LeGresley).