Euler's Equations of Motion in other coordinates

In cylindrical coordinates, (r, θ, z), Euler's equations of motion for an inviscid fluid become:

$$\rho \left[\frac{Du_r}{Dt} - \frac{u_\theta^2}{r} \right] = -\frac{\partial p}{\partial r} + f_r \tag{Bdc1}$$

$$\rho \left[\frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r} \right] = -\frac{1}{r}\frac{\partial p}{\partial \theta} + f_\theta \tag{Bdc2}$$

$$\rho \frac{Du_z}{Dt} = -\frac{\partial p}{\partial z} + f_z \tag{Bdc3}$$

where u_r, u_θ, u_z are the velocities in the r, θ, z directions, p is the pressure, ρ is the fluid density and f_r, f_θ, f_z are the body force components. The Lagrangian derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z} \tag{Bdc4}$$

For reference and completeness note that for an incompressible fluid the equation of continuity in cylindrical coordinates is

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \;=\; 0 \tag{Bdc5}$$

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In spherical coordinates, (r, θ, ϕ), Euler's equations of motion for an inviscid fluid become:

$$\rho \left\{ \frac{Du_r}{Dt} - \frac{u_\theta^2 + u_\phi^2}{r} \right\} = -\frac{\partial p}{\partial r} + f_r \tag{Bdc6}$$

$$\rho \left\{ \frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r} - \frac{u_\phi^2 \cot\theta}{r} \right\} = -\frac{1}{r}\frac{\partial p}{\partial \theta} + f_\theta \tag{Bdc7}$$

$$\rho \left\{ \frac{Du_\phi}{Dt} + \frac{u_\phi u_r}{r} + \frac{u_\theta u_\phi \cot\theta}{r} \right\} = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} + f_\phi \tag{Bdc8}$$

where u_r, u_θ, u_ϕ are the velocities in the r, θ, ϕ directions, p is the pressure, ρ is the fluid density and f_r, f_θ, f_ϕ are the body force components. The Lagrangian or material derivative is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + \frac{u_\phi}{r\sin\theta}\frac{\partial}{\partial \phi} \tag{Bdc9}$$

For completeness the equation of continuity for an incompressible fluid in spherical coordinates is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 u_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(u_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} = 0 \tag{Bdc10}$$