Navier-Stokes Equations in Cylindrical Coordinates

In cylindrical coordinates, (r, θ, z), the Navier-Stokes equations of motion for an incompressible fluid of constant dynamic viscosity, μ, and density, ρ, are

$$\rho \left[\frac{Du_r}{Dt} - \frac{u_\theta^2}{r} \right] = -\frac{\partial p}{\partial r} + f_r + \mu \left[\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} \right] \quad \text{(Bhg1)}$$

$$\rho \left[\frac{Du_\theta}{Dt} + \frac{u_\theta u_r}{r} \right] = -\frac{1}{r}\frac{\partial p}{\partial \theta} + f_\theta + \mu \left[\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} \right] \quad \text{(Bhg2)}$$

$$\rho \frac{Du_z}{Dt} = -\frac{\partial p}{\partial z} + f_z + \mu \nabla^2 u_z \quad \text{(Bhg3)}$$

where u_r, u_θ, u_z are the velocities in the r, θ, z cylindrical coordinate directions, p is the pressure, f_r, f_θ, f_z are the body force components in the r, θ, z directions and the operators D/Dt and ∇^2 are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r}\frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z} \quad \text{(Bhg4)}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad \text{(Bhg5)}$$