Effect of Suction

Sometimes fluid is removed by suction through a porous section of surface on, say, an airfoil in order to delay boundary layer separation. In this section we choose to show the effect of suction on a flat plate set parallel to a uniform stream of velocity, U. As sketched in Figure 1 we denote by V the velocity at which fluid is removed through the porous plate. In other words, the volume of fluid removed through the porous plate per unit plate length, per unit breadth (perpendicular to figure) and per unit time is equal to V.

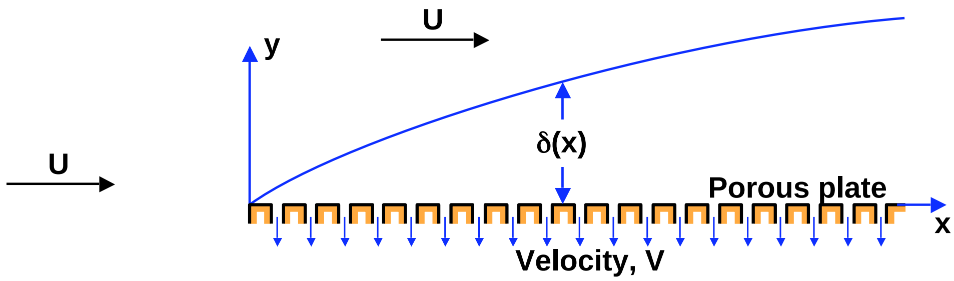

Figure 1: Boundary layer on a porous plate with suction.

The thickness of the boundary layer on the porous plate is denoted by $\delta(x)$. We will use approximate boundary layer methods assuming similarity of the velocity profile (in other words that $u/U = F(y/\delta)$ where the function F is not a function of x) to find a relation between the friction on the surface and the quantities V, U, $d\delta/dx$ and α where α is the profile parameter

$$\alpha = \int_0^1 F(1-F)d\left(\frac{y}{\delta}\right) \qquad \text{(Bjj1)}$$

We define a control volume between x and $x + dx$, bounded by the surface of the plate and extending to the edge of the boundary layer. Then continuity requires that the mass flow rate into the top of the control volume, M, is:

$$M = \frac{d}{dx}\left[\int_0^\delta \rho u dy\right] dx + \rho V dx \qquad \text{(Bjj2)}$$

Now, preparing to apply the momentum theorem in the x direction, the net flux of x-momentum out of the control volume in the x-direction, $\mathcal{M}$, is

$$\mathcal{M} = \frac{d}{dx}\left\{\int_0^\delta \rho u^2 dy\right\} dx - MU \qquad \text{(Bjj3)}$$

(Note that there is no x-direction momentum flux through the plate surface). After substituting for M this becomes

$$\mathcal{M} = \frac{d}{dx}\left\{\int_0^\delta \rho u^2 dy\right\} dx - \left[\frac{d}{dx}\left\{\int_0^\delta \rho u dy\right\} dx + \rho V dx\right] U$$

$$\mathcal{M} = \rho U^2 \left[-\frac{d}{dx}\left\{\int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy\right\} - \frac{V}{U}\right] dx$$

$$\mathcal{M} = \rho U^2 \left[-\alpha \frac{d\delta}{dx} - \frac{V}{U} \right] dx \tag{Bjj4}$$

Applying the momentum theorem we find that the force on the control volume in the x-direction, F_x, is:

$$\mathcal{M} = F_x = -\tau_W dx - \frac{dP}{dx}\delta(x)dx \tag{Bjj5}$$

and since there is no pressure gradient in the exterior flow and therefore no force due to a pressure difference it follows that

$$\tau_W = \rho U^2 \left[\alpha \frac{d\delta}{dx} + \frac{V}{U} \right] \tag{Bjj6}$$

The skin friction coefficient C_f is then given by:

$$C_f = \frac{\tau_W}{\frac{1}{2}\rho U^2} = 2 \left[\alpha \frac{d\delta}{dx} + \frac{V}{U} \right] \tag{Bjj7}$$

Thus the primary effect of the suction is to increase the skin friction through the $+V/U$ terms in equations (Bjj6) and (Bjj7) though this effect is somewhat offset by a reduction in $d\delta/dx$, an effect which can be shown to be secondary. Consequently the suction reduces the potential for flow separation since laminar boundary layer separation occurs when the friction decreases to zero. This effect is sometimes used on a portion of the surface of an airfoil in order to prevent or, at least, delay flow separation and thus improve the performance of the airfoil.