Quasistatic Stability Analyses

A quasistatic analysis of the stability of the equilibrium operating point is usually conducted in the following way. We consider perturbing the system to a new mass flow rate dm greater than that at the operating point as shown in Figure 1. Then, somewhat heuristically, one argues from Figure 1 that the total

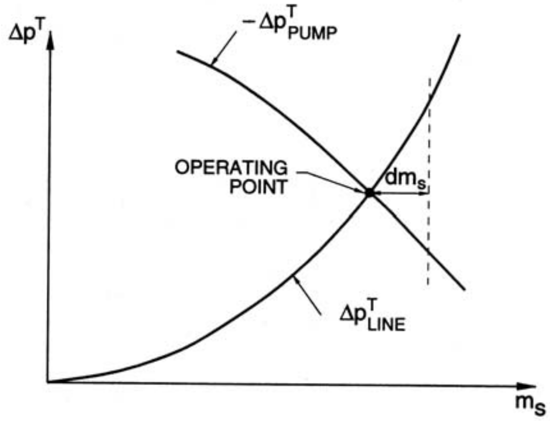

Figure 1: Alternate presentation of system equilibrium.

pressure rise across the *pumping* component is now less than the total pressure drop across the *pipeline* and therefore the flow rate will decline back to its value at the operating point. Consequently, the particular relationship of the characteristics in Figure 1 implies a stable operating point. If, however, the slopes of the two components are reversed (for example, Pump B of Figure 2(a) or the operating point C of Figure 2(b)) then the operating point is unstable since the increase in the flow has resulted in a *pump* total pressure that now exceeds the total pressure drop in the *pipeline*. These arguments lead to the conclusion that the operating point is stable when the slope of the system characteristic at the operating point is positive or

$$\frac{d\Delta p_s^T}{dm_s} > 0 \quad \textbf{or} \quad R_s^* > 0 \tag{Bnca1}$$

The same criterion can be derived in a somewhat more rigorous way by using an energy argument. Note that the net flux of flow energy out of each component is $m_k \Delta p_k^T$. In a straight pipe this energy is converted to heat through the action of viscosity. In a pump $m_k(-\Delta p_k^T)$ is the work done on the flow by the pump impeller. Thus the net energy flux out of the whole system is $m_s \Delta p_s^T$ and, at the operating point, this is zero (for simplicity we discuss a closed loop system) since $\Delta p_s^T = 0$. Now, suppose, that the flow rate is perturbed by an amount dm_s. Then, the new net energy flux out of the system is ΔE where

$$\Delta E = (m_s + dm_s)\left\{\Delta p_s^T + dm_s \frac{d\Delta p_s^T}{dm_s}\right\} \approx m_s dm_s \frac{d\Delta p_s^T}{dm_s} \tag{Bnca2}$$

Then we argue that if dm_s is positive and the perturbed system therefore dissipates more energy, then it must be stable. Under those circumstances one would have to add to the system a device that injected more energy into the system so as to sustain operation at the perturbed state. Hence the criterion (Bnca1) for quasistatic stability is reproduced.

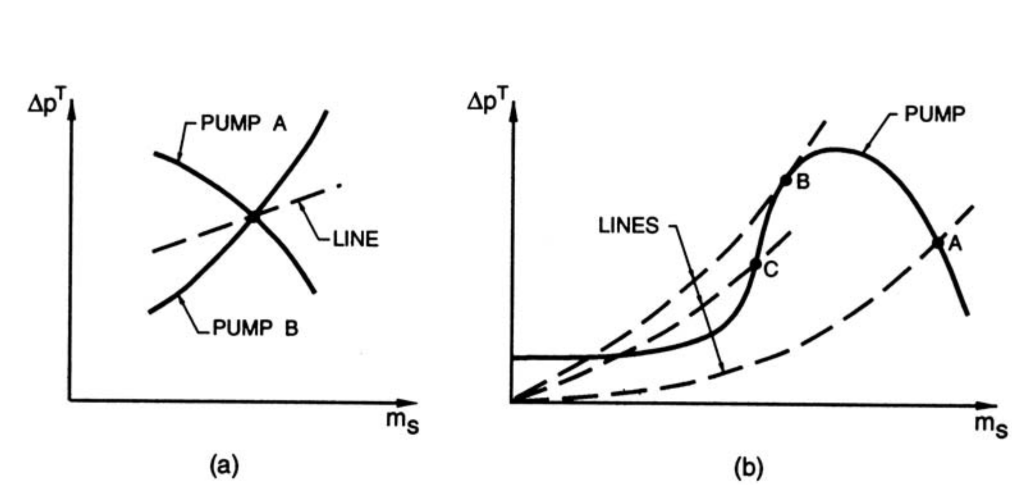

Figure 2: Quasistatically stable and unstable flow systems.