System with Rigid Body Vibration

All of the preceding analysis has assumed that the structure of the hydraulic system is at rest in some inertial coordinate system. However, there are a number of important problems in which the oscillation of the hydraulic system itself may play a central role. For instance, one might seek to evaluate the unsteady pressures and flow rates in a hydraulic system aboard a vehicle undergoing translational or rotational oscillations. Examples might be oil or water pumping systems aboard a ship, or the fuel and hydraulic systems on an aircraft. In other circumstances, the motion of the vehicle may couple with the propulsion system dynamics to produce instabilities, as in the simplest of the Pogo instabilities of liquid propelled rocket engines (see section (Mbfn)).

In this section we give a brief outline of how rigid body oscillations of the hydraulic system can be included in the frequency domain methodology. For convenience we shall refer to the structure of the hydraulic system as the "vehicle". There are, of course, more complex problems in which the deformation of the vehicle is important. Such problems require further refinement of the methods presented here.

In order to include the rigid body oscillation of the vehicle in the analysis, it is first necessary to define a coordinate system, $\underline{x}$, which is fixed in the vehicle, and a separate inertial or nonaccelerating coordinate system, $\underline{x}_A$. The mean location of the origin of the $\underline{x}$ system is chosen to coincide with the origin of the $\underline{x}_A$ system. The oscillations of the vehicle are then described by stating that the translational and rotational displacements of the $\underline{x}$ coordinate system in the $\underline{x}_A$ system are respectively given by

$$Re\left\{\underline{\tilde{d}}e^{j\omega t}\right\} \quad ; \quad Re\left\{\underline{\tilde{\theta}}e^{j\omega t}\right\} \qquad \text{(Bngk1)}$$

It follows that the oscillatory displacement of any vector point, $\underline{x}$, in the vehicle is given by

$$Re\left\{(\underline{\tilde{d}}+\underline{\tilde{\theta}}\times\underline{x})e^{j\omega t}\right\} \qquad \text{(Bngk2)}$$

and the oscillatory velocity of that point will be

$$Re\left\{j\omega(\underline{\tilde{d}}+\underline{\tilde{\theta}}\times\underline{x})e^{j\omega t}\right\} \qquad \text{(Bngk3)}$$

Then, if the steady and oscillatory velocities of the flow in the hydraulic system, and *relative to* that system, are given as in the previous sections by $\underline{\bar{u}}$ and $\underline{\tilde{u}}$ respectively, it follows that the oscillatory velocity of the flow in the nonaccelerating frame, $\underline{\tilde{u}}_A$, is given by

$$\underline{\tilde{u}}_A = \underline{\tilde{u}} + j\omega(\underline{\tilde{d}}+\underline{\tilde{\theta}}\times\underline{x}) \qquad \text{(Bngk4)}$$

Furthermore, the acceleration of the fluid in the nonaccelerating frame, $\underline{\tilde{a}}_A$, is given by

$$\underline{\tilde{a}}_A = j\omega\underline{\tilde{u}} - \omega^2\underline{\tilde{d}} - \omega^2\underline{\tilde{\theta}}\times\underline{x} + 2j\omega\underline{\tilde{\theta}}\times\underline{\bar{u}} \qquad \text{(Bngk5)}$$

The last three terms on the right hand side are vehicle-induced accelerations of the fluid in the hydraulic system. It follows that these accelerations will alter the difference in the total pressure between two nodes of the hydraulic system denoted by subscripts 1 and 2. By integration one finds that the total pressure difference, $(\tilde{p}_2^T - \tilde{p}_1^T)$, is related to that which would pertain in the absence of vehicle oscillation, $(\tilde{p}_2^T - \tilde{p}_1^T)_0$, by

$$(\tilde{p}_2^T - \tilde{p}_1^T) = (\tilde{p}_2^T - \tilde{p}_1^T)_0 + \rho\omega^2\left\{(\underline{x}_2 - \underline{x}_1)\cdot\underline{\tilde{d}} + (\underline{x}_2\times\underline{x}_1)\cdot\underline{\tilde{\theta}}\right\} \qquad \text{(Bngk6)}$$

where $\underline{x}_2$ and $\underline{x}_1$ are the locations of the two nodes in the frame of reference of the vehicle.

The inclusion of these acceleration-induced total pressure changes is the first step in the synthesis of models of this class of problems. Their evaluation requires the input of the location vectors, $\underline{x}_i$, for each of the system nodes, and the values of the system displacement frequency, ω, and amplitudes, $\underline{\tilde{d}}$ and $\underline{\tilde{\theta}}$. In an analysis of the response of the hydraulic system, the vibration amplitudes, $\underline{\tilde{d}}$ and $\underline{\tilde{\theta}}$, would be included as inputs. In a stability analysis, they would be initially unknown. In the latter case, the system of equations would need to be supplemented by those of the appropriate feedback mechanism. An example would be a set of equations giving the unsteady thrust of an engine in terms of the fluctuating fuel supply rate and pressure and giving the accelerations of the vehicle resulting from that fluctuating thrust. Clearly a complete treatment of such problems would be beyond the scope of this book.