Frequency Domain Methods

When the quasistatic assumption for a device like a pump or turbine becomes questionable, or when the complexity of the fluid or the geometry makes the construction of a set of differential equations impractical or uncertain, then it is clear that experimental information on the dynamic behavior of the device is necessary. In practice, such experimental information is most readily obtained by subjecting the device to fluctuations in the flow rate or head for a range of frequencies, and measuring the fluctuating quantities at inlet and discharge. Such experimental results will be presented later. For present purposes it is sufficient to recognize that one practical advantage of frequency domain methods is the capability of incorporation of experimentally obtained dynamic information and the greater simplicity of the experiments required to obtain the necessary dynamic data. Another advantage, of course, is the core of fundamental knowledge that exists regarding such methodology (see for example, Pipes 1940, Hennyey 1962, Paynter 1961, Brown 1967). As stated earlier, the disadvantage is that the methods are limited to small linear perturbations in the flow rate. When the perturbations are linear, Fourier analysis and synthesis can be used to convert from transient data to individual frequency components and vice versa. All the dependent variables such as the mean velocity, u, mass flow rate, m, pressure, p, or total pressure, p^T, are expressed as the sum of a mean component (denoted by an overbar) and a complex fluctuating component (denoted by a tilde) at a frequency, ω, which incorporates the amplitude and phase of the fluctuation:

$$p(s,t) = \bar{p}(s) + Re\left\{\tilde{p}(s,\omega)e^{j\omega t}\right\} \qquad \text{(Bnga1)}$$

$$p^T(s,t) = \bar{p}^T(s) + Re\left\{\tilde{p}^T(s,\omega)e^{j\omega t}\right\} \qquad \text{(Bnga2)}$$

$$m(s,t) = \bar{m}(s) + Re\left\{\tilde{m}(s,\omega)e^{j\omega t}\right\} \qquad \text{(Bnga3)}$$

where j is $(-1)^{\frac{1}{2}}$ and Re denotes the real part. Since the perturbations are assumed linear ($|\tilde{u}| \ll \bar{u}, |\tilde{m}| \ll \bar{m}$, etc.), they can be readily superimposed, so a summation over many frequencies is implied in the above expressions. In general, the perturbation quantities will be functions of the mean flow characteristics as well as position, s, and frequency, ω.

We should note that there do exist a number of codes designed to examine the frequency response of hydraulic systems using frequency domain methods (see, for example, Amies and Greene 1977).