Distributed Systems

In the case of a distributed system such as a pipe, it is also appropriate to define a matrix $[F]$ (see Brown 1967) so that

$$\frac{d}{ds}\{\tilde{q}^n\} = -[F(s)]\{\tilde{q}^n\} \tag{Bngd1}$$

Note that, apart from the frictional term, the equations (Gbbb7) and (Gbbb8) for flow in a pipe will lead to perturbation equations of this form. Furthermore, in many cases the frictional term is small, and can be approximated by a linear term in the perturbation equations; under such circumstances the frictional term will also fit into the form given by equation (Bngd1).

When the matrix $[F]$ is independent of location, s, the distributed system is called a "uniform system" (see section entitled "Properties of Transfer Matrices"). For example, in equations (Gbbb7) and (Gbbb8), this would require ρ, c, a, f and A_0 to be approximated as constants (in addition to the linearization of the frictional term). Under such circumstances, equation (Bngd1) can be integrated over a finite length, ℓ, and the transfer matrix $[T]$ of the form (Bngc6) becomes

$$[T] = e^{-[F]\ell} \tag{Bngd2}$$

where $e^{[F]\ell}$ is known as the "transmission matrix." For a system of order two, the explicit relation between $[T]$ and $[F]$ is

$$T_{11} = jF_{11}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) + \left(\lambda_2 e^{-j\lambda_1\ell} - \lambda_1 e^{-j\lambda_2\ell}\right)/(\lambda_2-\lambda_1)$$

$$T_{12} = jF_{12}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1)$$

$$T_{21} = jF_{21}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1)$$

$$T_{22} = jF_{22}\left(e^{-j\lambda_2\ell} - e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) + \left(\lambda_2 e^{-j\lambda_2\ell} - \lambda_1 e^{-j\lambda_1\ell}\right)/(\lambda_2-\lambda_1) \tag{Bngd3}$$

where λ_1, λ_2 are the solutions of the equation

$$\lambda^2 + j\lambda(F_{11}+F_{22}) - (F_{11}F_{22} - F_{12}F_{21}) = 0 \tag{Bngd4}$$

Some special features and properties of these transfer functions will be explored in the sections which follow.