Turbulence Models

In order to solve the equations for turbulent flow it is necessary to find relations for the Reynolds stresses in terms of the mean flow properties, specifically the mean flow velocities. Many such models have been proposed over the years and we will only give a brief review here. It is appropriate to begin with Prandtl's original idea, known as Prandtl's mixing length model. Focusing on a typical boundary layer near a wall as

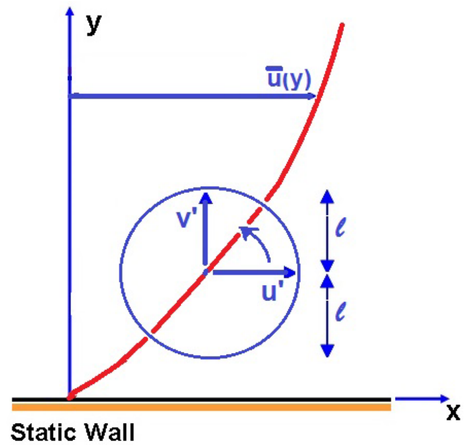

Figure 1: Notation for Prandtl's mixing length model.

shown in Figure 1 and on a typical turbulent eddy of size, ℓ, embedded in that layer, Prandtl argued that fluid with typical x velocity, u, closer to the wall would be transported to a position a distance ℓ (known as Prandtl's mixing length) more distant from the wall where the local mean velocity is $u + \ell(du/dy)$. If it carried with it the original velocity u then a typical local velocity fluctuation at that further position would be $\ell(du/dy)$ and so a reasonable estimate for u' (or v') would be $\ell(du/dy)$. Consequently Prandtl argued that the Reynolds shear stress might be given by

$$\overline{u'v'} \approx \ell^2 \left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{(Bkh1)}$$

The second step is to estimate ℓ and since the Reynolds shear stress must be zero at the wall it seemed appropriate to set $\ell = \kappa y$ where κ was some constant (ambitiously termed the "universal constant") for which the experimental data suggest the not unreasonable value of about 0.4. This yields

$$\overline{u'v'} \approx \kappa^2 y^2 \left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{(Bkh2)}$$

We will use this classic model in several later constructions.