# Viscous Losses in Incompressible Fluid Flows

Though Bernoulli's equation applies to inviscid flow, we may, in general terms, incorporate the effects of viscosity by recognizing that viscous effects will cause mechanical energy to be dissipated into heat so that the total head or total pressure, instead of remaining constant, will decrease in the direction of flow (provided some actice surface is not injecting energy into the flow). Consequently in the flow of a viscous fluid through a passive component like a duct or valve the discharge (subscript 2) total head will be less than the inlet (subscript 1) total head:

$$H_1 - H_2 \; = \; \left\{ \frac{p}{(\rho g)} + \frac{|\underline{u}|^2}{(2g)} + y \right\}_1 \; - \; \left\{ \frac{p}{(\rho g)} + \frac{|\underline{u}|^2}{(2g)} + y \right\}_2 \; = \; \Delta H \; > \; 0 \qquad \text{(Bfb1)}$$

Clearly one of the most common tasks that a fluid engineer must face is the evaluation of these losses, $\Delta H$, in a pipe or duct system and there exist many compendiums listing these losses in a huge array of devices, components and fittings (see, for example, Moody (1944), Idelchik (1994), Crane (1957)).

In the present text, we give only a brief conceptual overview of these loss mechanisms. For this purpose

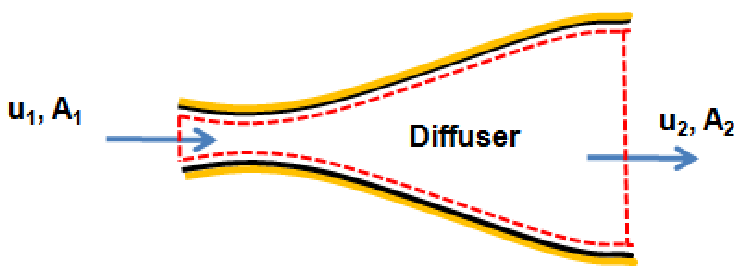


Figure 1: A diffuser with steady, incompressible flow.

it is useful to begin with a discussion of one of the simplest devices, namely a diffuser, a length of duct in which the cross-sectional area is increasing in the direction of flow as depicted in Figure 1. In the absence of viscous losses and if the increase in cross-sectional area ($A_2 > A_1$) is sufficiently gradual so that the flow at both inlet and discharge have a uniform velocity then in steady flow Bernouilli's equation would yield $H_2 = H_1$. Since by continuity $A_2 u_2 = A_1 u_1$and therefore $u_2 < u_1$, it follows that $p_2$ would be greater than $p_1$ (assuming the diffuser is horizontal so that $y_2 = y_1$). This phenomena of an increase in pressure in the direction of flow is known as **pressure recovery**; indeed, diffusers are frequently deployed in order to convert dynamic head or velocity into pressure or static head, for example at the discharge from a pump. Specifically, using the continuity equation $A_2 u_2 = A_1 u_1$ it follows from Bernoulli's equation that, in the absence of viscous effects (and assuming $y_2 = y_1$):

$$p_2 - p_1 \; = \; \frac{\rho u_1^2}{2} \left\{ 1 - \frac{A_1^2}{A_2^2} \right\} \quad \text{and} \quad \Delta H \; = \; 0 \qquad \text{(Bfb2)}$$

If this were the case we could call the diffuser performance "perfect". If, on the other hand, the viscous effects consumed all of the possible pressure recovery then $p_2 = p_1$, the pressure recovery is zero and, by the definition of the losses, $\Delta H$ (equation (Bfb1)), is given by

$$\Delta H = \; \frac{u_1^2}{2g} \left\{ 1 - \frac{A_1^2}{A_2^2} \right\} \qquad \text{(Bfb3)}$$

In practice, when the viscous effects are significant but not all-consuming, the loss, $\Delta H$ lies somewhere between zero and the expression in equation (Bfb3). This implies that the viscous loss is some fraction of the dynamic head, $u_1^2/2g$. It can be shown that this it is almost always the case and that the viscous loss is proportional to $U^2/2g$ where $U$ is a characteristic velocity of the flow. Consequently, in general, the head loss is almost always non-dimensionalized using that dynamic head, $U^2/2g$, thus defining a loss coefficient, $K$, given by

$$K = \frac{\Delta H}{\left\{\frac{U^2}{2g}\right\}} \qquad \text{(Bfb4)}$$

On the associated pages we survey the loss coefficients for a range of different fittings and components including pipes, nozzles, diffusers, fittings and valves.