Rocket Engines

In section (Bof) we discussed how a steady, one-dimensional isentropic flow flow could become supersonic as it passes through a throat and into a diffuser. In such a flow, the velocity and Mach number will continue to increase as long as the area increases and the flow remains isentropic. The only way that this acceleration can be bounded is if the flow encounters a shock wave in the diffuser so that the flow downstream reverts to subsonic. This section explores how and where this happens and we do so in the context of what is known as a *de Laval nozzle* which is simply a converging/diverging nozzle supplied from a large reservoir as depicted in Figure 1. This is also a basic rocket engine in which the hot gas from the

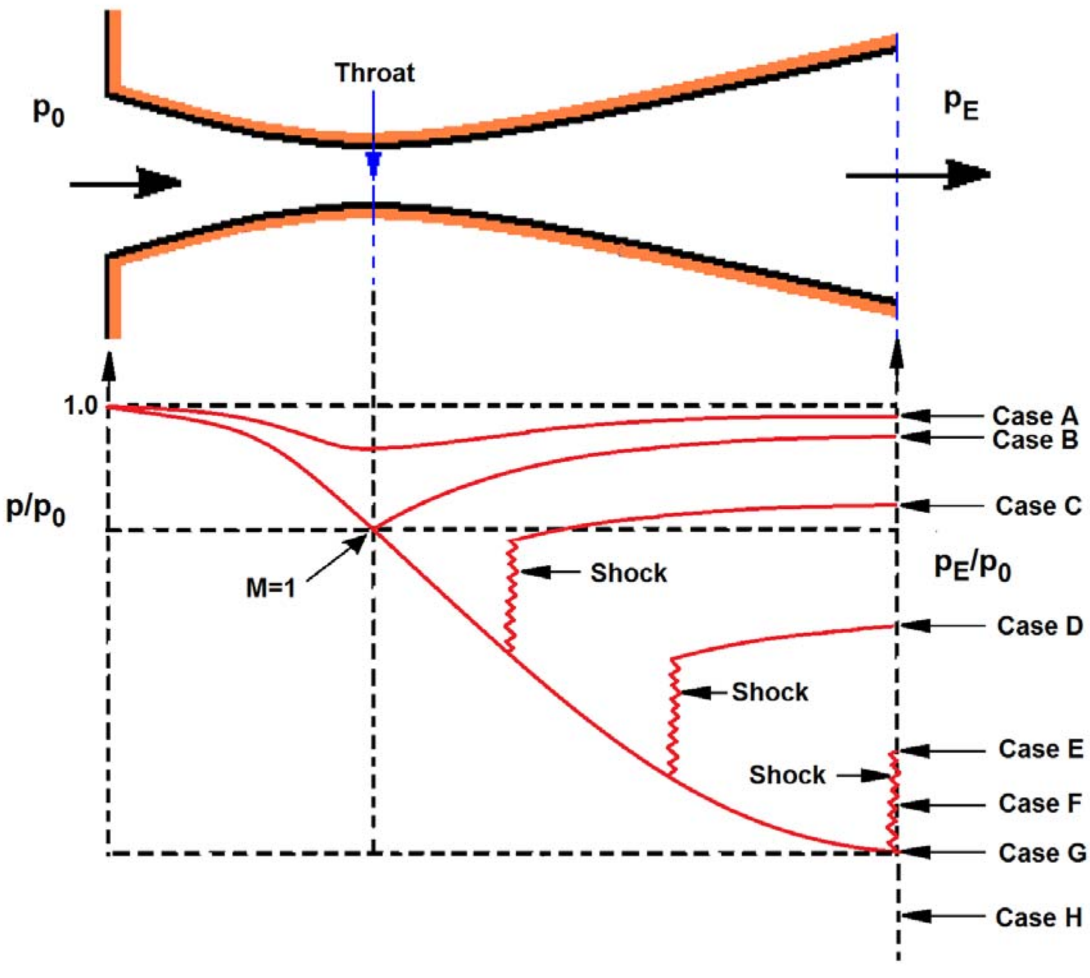

Figure 1: A de Laval nozzle with the pressure distributions for various exit pressures.

reservoir at high pressure is accelerated through a throat to supersonic speeds that continue to increase in the diffuser so as to maximize the thrust the device produces. However, a shock wave in the diffuser would reduce the thrust and the performance of the engine and so the design should be such as to minimize the potential formation of a shock. We explore how this is achieved.

The analysis is most instructively pursued by keeping the reservoir pressure, p_0, fixed and exploring the development of the flow as the external pressure downstream of the diffuser, p_E, is decreased starting at

$p_E = p_0$ when the flow is zero. The developments will proceed through a series of typical flows that are exemplified by the Cases A through G shown in Figure 1. The progression of developments are as follows:

- Case A: For pressures, p_E, just below p_0 the flow through the duct will be subsonic and the throat will not be choked. The exit pressure, p_E, will be close to the atmospheric pressure, p_A, in the environment outside of the emerging jet and the flow rate will be proportional to the pressure difference, $p_0 - p_A$.
- Case B: The flow in the throat reaches $M = 1$ but continues subsonic throughout the diffuser. The flow at this point is choked (given by equation (Bof4)) and the pressure upstream of the throat and the mass flow rate are the same for all subsequent cases. However the thrust is greater the greater the jet velocity at the exit.
- Cases C and D: At exit pressures below that of Case B, the flow downstream of the throat is supersonic but a shock wave forms in the diffuser converting the flow to subsonic. Otherwise the flows upstream and downstream of the shock are isentropic. We note that if the exit pressure, p_E, is increased again the shock wave moves back up toward the throat but weakens as it does so that its strength tends to zero as the throat is approached. On the other hand, as p_E is decreased the shock wave approaches the diffuser exit. Note also that since the frictional effects at the walls are assumed negligible all the variations apart from those across the shock are isentropic and along those lines the total pressure, p_0, remains constant. However the total pressure upstream of the shock, p_{01}, is different from that downstream of the shock, p_{02}.
- Case E: The shock has reached the exit but the flow emerging is still subsonic.
- Case F: As the pressure p_E is decreased below that of Case E, the shock is pushed out of the end of the diffuser and a complex, three-dimensional flow forms downstream of the diffuser exit. We address this flow in a later section. For reasons described below a state such as Case F is referred to as an *over-expanded* flow. It would have been *correctly expanded* (see below) if the diffuser had ended at a position where the exit area was smaller.
- Case G: At this exit pressure the supersonic flow emerges from the diffuser with a pressure, p_E, equal to the atmospheric pressure, p_A, outside the jet. This is referred to as a *correctly expanded* flow and represents the optimum for rocket engine thrust, combining the fixed mass flow rate with the highest emerging velocity of flow.
- Case H: If the exit pressure is further decrease to values lower than Case G and lower than p_A another complex three-dimensional flow is created that involves the jet transitioning back to p_A. This is referred to as an *under-expanded* flow since an extension of the diffuser to a larger exit area could recover a correctly expanded condition. As with Case F, the three-dimensional flow generated by an *under-expanded* diffuser will be addressed in a later section.

It is appropriate to revisit some of these cases to indicate how a typical calculation might progress:

[1] The first step is to evaluate the circumstances under which the flow in the throat becomes choked. At this point (and for all cases after Case B) the throat pressure, p^*, is given by equation (Boe8), namely

$$\frac{p^*}{p_0} = \left\{\frac{2}{(\gamma+1)}\right\}^{\gamma/(\gamma-1)} \quad \text{(Boi1)}$$

and the mass flow rate for this and all later cases is given by equation (Bof4). The next step is to evaluate the isentropic flow downstream of the throat. Given the area ratio, A_E/A^*, where A_E is the area of the diffuser exit and, as usual A^* is the throat area, we can determine the exit Mach number, M_E, from equation (Boe12) (or from the table, Figure 1 in section (Boe)) and then determine p_E/p^*

from equation (Boe10) or the same table. This allows evaluation of the exit pressure ratio p_E/p_0 for Case B and therefore the pressure at which the flow becomes choked.

[2] Wherever it is located in the diffuser, the upstream total pressure for the shock is simply $p_{01} = p_0$. Then the next step is to determine the location of the shock in the diffuser and this is best addressed iteratively. Suppose that we use a guessed location and cross-sectional area, A_S, for the shock. Then, given A_S/A^*, the upstream Mach number, M_1, follows from equation (Boe12) and the downstream Mach number from equation (Boh9) (or the table, Figure 1 from section (Boh)). The pressures upstream and downstream of the shock also follow. Moreover knowing M_1 the ratio of the total pressures, p_{02}/p_{01}, and therefore p_{02} can be determined.

[3] Then, moving to the flow downstream of the shock, knowing the area ratio A_S/A_E and M_2 permits evaluation of the Mach number at the exit from the diffuser, M_E, and the pressure at the exit p_E. If this is not the required exit pressure, p_E, then the location of the shock needs to be adjusted until the calculated p_E matches the desired p_E.

[4] Case G is relatively easily determined. Knowing A_E/A^*, equation (Boe12) (or the isentropic table) can be used to determine the supersonic M_E and the pressure ratio, p_E/p^*, (and therefore p_E) follows from equation (Boe10) (or the isentropic table).

[5] Case E follows from Case G by another application of the relations across a normal shock wave.