Exact Solutions to the Laminar Boundary Layer Equations

The equations for the steady, planar, incompressible, Newtonian flow in a laminar boundary layer bounded by an external irrotational flow with velocity, $U(s)$, namely

$$\frac{\partial u}{\partial s} + \frac{\partial v}{\partial n} = 0 \tag{Bjc1}$$

$$u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} = U\frac{dU}{ds} + \nu\frac{\partial^2 u}{\partial n^2} \tag{Bjc2}$$

have only a few exact solutions since they represent a set of complicated, non-linear equations. We focus on those few exact solutions and the useful results they yield.

The exact solutions that have been found pertain to simple, polynomial forms of the input function $U(s)$. The first, discovered by Blasius (1908), was for a simple constant U and pertains to the case of a uniform stream of velocity, U, encountering an infinitely thin flat plate set parallel with that stream. We detail that solution in the following section (Bjd). Years later Falkner and Skan discovered a more general set of solutions for the following simple forms of $U(s)$:

$$U(s) = Cc^m \qquad \text{so that} \qquad U\frac{dU}{ds} \;=\; m\; C^2\; s^{2m-1} \tag{Bjc3}$$

where C and m are constants.

These "exact solutions" are valuable in understanding the properties and behavior of laminar boundary layers in general and are extensively used in the construction of methods to solve for laminar boundary layers with more complicated external flow velocites, $U(s)$. We therefore examine both the Blasius and Falkner-Skan solutions in the next two sections. Following that we will turn our attention to approximate methods of solution for the laminar boundary layer equations.

Finally, we should recall that while the Blasius and Falkner-Skan solutions are exact solutions to the laminar boundary layer equations they are *not* exact solutions to the Navier-Stokes equations since the laminar boundary layer equations are approximate versions of the Navier-Stokes equations.