Viscous Effects in Linear Cascades

It is also of value to examine in more detail the mechanism of viscous loss in a cascade. Even in two-dimensional cascade flow, the growth of the boundary layers on the pressure and suction surfaces of the blades, and the wakes they form downstream of the blades (see figure 1), are complex, and not amenable to simple analysis. However, as the reviews by Roudebush and Lieblein (1965) and Lieblein (1965) demonstrate, it is nevertheless possible to provide some qualitative guidelines for the resulting viscous effects on cascade performance. In this respect, the diffusion factor, introduced by Lieblein *et al.* (1953), is a useful concept that is based on the following approximations. First, we note that under normal operating conditions, the boundary layer on the suction surface will be much thicker than that on the pressure surface of the foil, so that, to a first approximation, we may neglect the latter. Then, the thickness of the wake (and therefore the total pressure loss) will be primarily determined by that fraction of the suction surface over which the velocity gradient is adverse, since that is where the majority of the boundary layer growth occurs. Therefore, Lieblein *et al.* argued, the momentum thickness of the wake, θ^*, should correlate with a parameter they termed the *diffusion factor*, given by $(w_{max} - w_2)/w_{max}$, where w_{max} is the maximum velocity on the suction surface. One should visualize deceleration or diffusion of the flow from w_{max} to w_2, and that this diffusion is the primary factor in determining the wake thickness. However, since w_{max} is not easily determined, Lieblein *et al.* suggest an approximation to the diffusion factor that is denoted Df, and given by

$$Df = 1 - \frac{w_2}{w_1} + \frac{v_{\theta 2} - v_{\theta 1}}{2sw_1}$$

$$Df = 1 - \frac{\sin\beta_1}{\sin\beta_2} + \frac{\sin\beta_1(\cot\beta_1 - \cot\beta_2)}{2s} \quad \text{(Mbcd1)}$$

Figure 2 shows the correlation of the momentum thickness of the wake (normalized by the chord) with this diffusion factor, Df, for three foil profiles. Such correlations are now commonly used to determine the viscous loss due to blade boundary layers and wakes. Note that, once θ^*/c has been determined from such a correlation, the drag coefficient, C_D, and the friction or loss coefficient follow from equations (Mbcb7), (Mbcb9) and (Mbcb10) and the fact that $D = \rho w_2^2 \theta^*$:

$$C_D = \frac{2\sin^2\beta_M}{\sin^2\beta_2}\frac{\theta^*}{c} \quad ; \quad f = \frac{s\sin\beta_M}{\sin^2\beta_2}\frac{\theta^*}{c} \quad \text{(Mbcd2)}$$

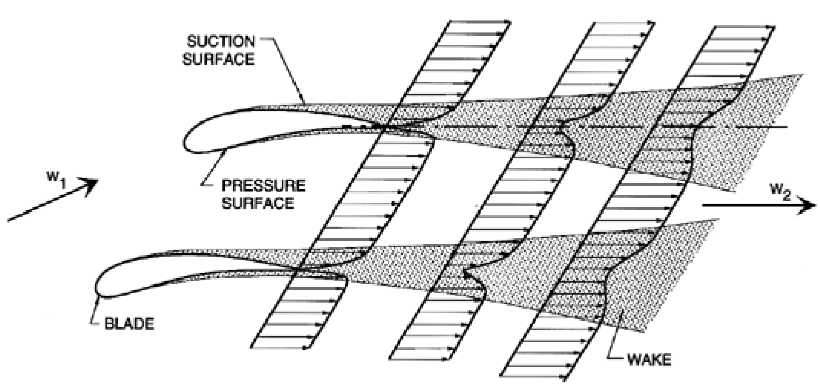

Figure 1: Sketch of the boundary layers on the surfaces of a cascade and the resulting blade wakes.

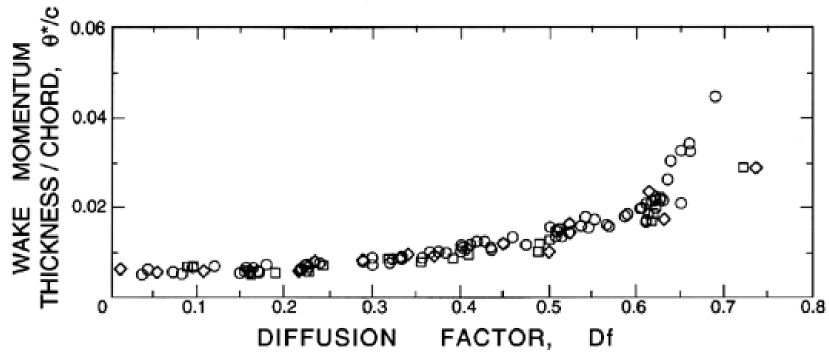

Figure 2: Correlation of the ratio of the momentum thickness of the blade wakes, θ^*, to the chord, c, with the diffusion factor, Df, for cascades of blades with three different profiles: NACA $65-(A_{10})10$ series (○) and two British C.4 parabolic arc profiles (□ and ◇). The maximum thickness of the blades is $0.1c$ and the Reynolds number is 2.5×10^5. Adapted from Lieblein (1965).

The data shown in figure 2 were for a specific Reynolds number, Re, and the correlations must, therefore, be supplemented by a statement on the variation of the loss coefficient with Re. A number of correlations of this type exist (Roudebush and Lieblein 1965), and exhibit the expected decrease in the loss coefficient with increasing Re. For more detail on viscous losses in a cascade, the reader should consult the aforementioned papers by Lieblein.

In an actual turbomachine, there are several additional viscous loss mechanisms that were not included in the cascade analyses discussed above. Most obviously, there are additional viscous layers on the inner and outer surfaces that bound the flow, the hub and the shroud (or casing). These often give rise to complex, three-dimensional secondary flows that lead to additional viscous losses (Horlock and Lakshminarayana 1973). Moreover, the rotation of other, "non-active" surfaces of the impeller will lead to viscous shear stresses, and thence to losses known as "disk friction losses" in the terminology of turbomachines. Also, leakage flows from the discharge back to the suction, or from one stage back to a preceding stage in a multistage pump, constitute effective losses that must be included in any realistic evaluation of the losses in an actual turbomachine (Balje 1981).