Radial Cascade Analyses

Two-dimensional models for centrifugal or radial turbomachines begin with analyses of the flow in a radial cascade (section (Mbbb) and figure 1), the counterpart of the linear cascade for axial flow machines. More specifically, the counterpart of the linear flat plate cascade is the logarithmic spiral cascade, defined in section (Mbbb), and shown in more detail in figure 1. There exist simple conformal mappings that allow potential flow solutions for the linear cascade to be converted into solutions for the corresponding radial cascade flow, though the proper interpretation of these solutions requires special care. The resulting head/flow characteristic for frictionless flow in a radial cascade of infinitely thin logarithmic spiral blades is given in a classic paper by Busemann (1928), and takes the form

$$\psi = Sf_B - \psi_0\phi\left(\cot\beta_b + \frac{v_{\theta 1}}{v_{m1}}\right) \tag{Mbce1}$$

The terms Sf_B and ψ_0 result from quite separate and distinct fluid mechanical effects. The term involving ψ_0 is a consequence of the frictionless, potential flow head rise through any simple, nonrotating cascade whether of axial, radial, or mixed flow geometry. Therefore, ψ_0 is identical to the quantity, ψ_0, defined by equation (Mbcb17) in the context of a linear cascade. The values for ψ_0 for a simple cascade of infinitely thin blades, whether linear, radial or mixed flow, are as given in figure 1, section (Mbcc). The ψ_0 term can be thought of as the "through flow" effect, and, as demonstrated by figure 1, section (Mbcc), the value of ψ_0 rapidly approaches unity when the solidity increases to a value a little greater than one.

However, it is important to recognize that the ψ_0 term is the result of a frictionless, potential flow solution in which the vorticity is zero. This solution would be directly applicable to a static or nonrotating

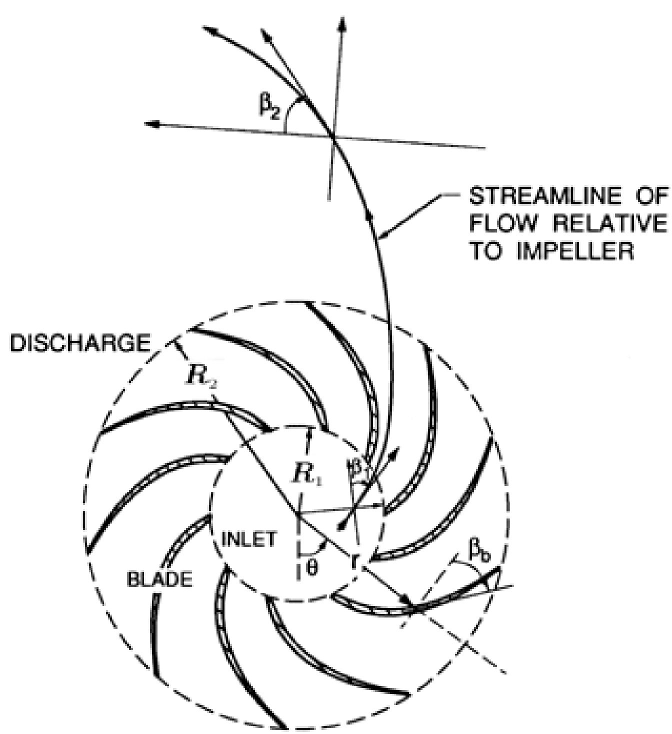

Figure 1: Schematic of the radial cascade corresponding to the linear cascade of figure ??.

radial cascade in which the flow entering the cacade has no component of the vorticity vector in the axial direction. This would be the case for a nonswirling axial flow that is deflected to enter a nonrotating, radial cascade in which the axial velocity is zero. But, relative to a *rotating* radial cascade (or centrifugal pump impeller), such an inlet flow does have vorticity, specifically a vorticity with magnitude 2Ω and a direction of rotation opposite to the direction of rotation of the impeller. Consequently, the frictionless flow through the impeller is not irrotational, but has a constant and uniform vorticity of -2Ω.

In inviscid fluid mechanics, one frequently obtains solutions for these kinds of rotational flows in the following way. First, one obtains the solution for the irrotational flow, which is represented by ψ_0 in the current problem. Mathematically, this is the complementary solution. Then one adds to this a particular solution that satisfies all the same boundary conditions, but has a uniform vorticity, -2Ω. In the present context, this particular, or rotational, solution leads to the term, Sf_B, which, therefore, has a quite different origin from the irrotational term, ψ_0. The division into the rotational solution and the irrotational solution is such that all the net volumetric flow through the impeller is included in the irrotational (or ψ_0) component. The rotational solution has no through flow, but simply consists of a rotation of the fluid within each blade passage, as sketched in figure 2. Busemann (1928) called this the

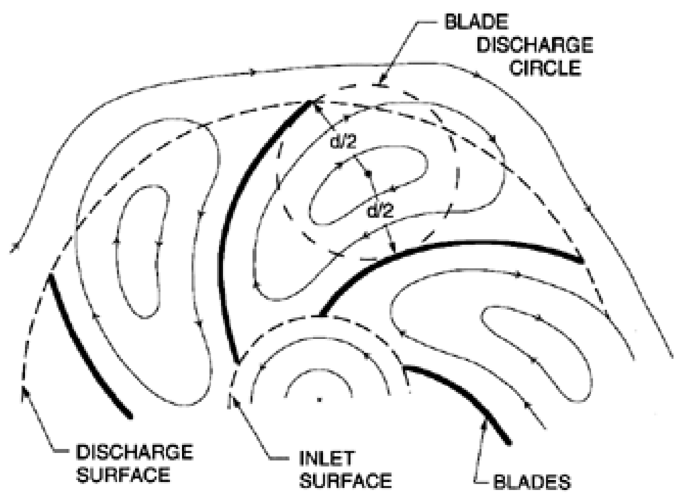

Figure 2: A sketch of the displacement component of the inviscid flow through a rotating radial cascade.

displacement flow; other authors refer to its rotating cells as *relative eddies* (Balje 1980, Dixon 1978). In his pioneering work on the fluid mechanics of turbomachines, Stodola (1927) was among the first to recognize the importance of this rotational component of the solution. Busemann (1928) first calculated its effect upon the head/flow characteristic for the case of infinitely thin, logarithmic spiral blades, in other words the simple cascade in the radial configuration. For reasons which will become clear shortly, the function, Sf_B, is known as the Busemann slip factor, and Busemann's solutions lead to the values presented in figure 3 when the solidity, $s > 1.1$. Note that the values of Sf_B are invariably less than or equal to unity, and, therefore, the effect of the displacement flow is to cause a decrease in the head. This deficiency can, however, be minimized by using a large number of blades. As the number of blades gets larger, Sf_B tends to unity as the rotational flow within an individual blade passage increasingly weakens. In practice, however, the frictional losses will increase with the number of blades. Consequently, there is an important compromise that must be made in choosing the number of blades. As figure 3 shows, this compromise will depend on the blade angle. Furthermore, the compromise must also take into account the structural requirements for the blades. Thus, radial machines for use with liquids usually have a smaller number of blades than those used for gases. The reason for this is that a liquid turbomachine requires much thicker blades, and, therefore, each blade creates much more flow blockage than in the case of a gas turbomachine. Consequently, liquid machines tend to have a smaller number of blades, typically eight for the range of

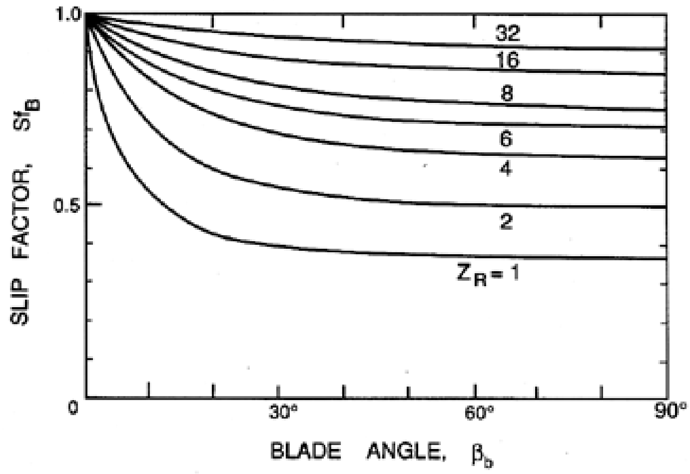

Figure 3: The Busemann slip factor, Sf_B, plotted against the blade angle, β_b, for various numbers of blades, Z_R. The results shown are for radial cascades of infinitely thin logarithmic spiral blades with solidities, $s > 1.1$. Adapted by Sabersky, Acosta and Hauptmann (1989) and Wislicenus (1947) from Busemann's (1928) theory.

specific speeds for which radial machines are designed ($N_D < 1.5$) (Stepanoff 1948, Anderson). Another popular engineering criterion (Stepanoff 1948) is that Z_R should be one third of the discharge blade angle, β_b (in degrees).

The decrease in the head induced by the displacement flow is due to the nonuniformity in the discharge flow; this nonuniformity results in a *mean* angle of discharge (denoted by β_2) that is different from the discharge blade angle, β_{b2}, and, therefore, implies an effective deviation angle or *slip*, Sf (see section (Mbba)). In fact, it is clear that the relations (Mbbc4), (Mbbg3), (Mbce1) and (Mbce3) imply that $Sf = Sf_B$, and, hence, the terminology used above. Stodola (1927) recognized that slip would be a consequence of the displacement flow, and estimated the magnitude of the slip velocity, $v_{\theta s}$, in the following approximate way. He argued that the slip velocity could be roughly estimated as $\Omega d/2$, where $d/2$ is the radius of the *blade discharge circle* shown in figure 2. He visualized this as representative of the rotating cell of fluid in a blade passage, and that the rotation of this cell at Ω would lead to the aforementioned $v_{\theta s}$. Then, provide Z_R is not too small, $d \approx 2\pi R_2 \sin\beta_{b2}$, and it follows that

$$v_{\theta s} = \pi \Omega R_2 \sin\beta_{b2}/Z_R \tag{Mbce2}$$

and, from equation (Mbba4), that the estimated slip factor, Sf_S, is

$$Sf_S = 1 - \frac{\pi \sin\beta_{b2}}{Z_R} \tag{Mbce3}$$

Numerical comparisons with the more exact results of Busemann presented in figure 3, show that equation (Mbce3) gives a reasonable first approximation. For example, an impeller with four blades, a blade angle of 25°, and a solidity greater than unity, has a Stodola slip factor of $Sf_S = 0.668$ compared to the value of $Sf_B = 0.712$ from Busemann's more exact theory.

There is a substantial literature on slip factors for centrifugal pumps. Some of this focuses on the calculation of slip factors for inviscid flow in radial cascades with blades that are more complex than the infinitely thin, logarithmic spiral blades used by Busemann. Useful reviews of some of this work can be found, for example, in the work of Wislicenus (1947), Stanitz (1952), and Ferguson (1963). Other researchers attempt to find slip factors that provide the best fit to experimental data. In doing so, they also attempt to account for viscous effects in addition to the inviscid effect for which the slip factor was originally devised. As an example of this approach, the reader may consult Wiesner (1967), who reviews the existing, empirical slip factors, and suggests one that seems to yield the best comparison with the experimental measurements.